Finding the Vertex of a Parabola

Using the Vertex Formula: \( x = -\frac{b}{2a} \)

The vertex formula is one method for determining the vertex of a parabola. Recall that a parabola is formed when graphing a **quadratic equation**. The parabola will normally present with both ends heading up, or with both ends heading down, as seen below. To use the vertex formula, a **quadratic equation** must be put in the form \( f(x) = ax^2 + bx + c \), where \( a \neq 0 \). We will use \( f(x) = x^2 - 4x - 3 \) as an example.

To find the coordinates of the vertex using \( x = -\frac{b}{2a} \)

1) Put the equation in proper form.
   \[ f(x) = x^2 - 4x - 3 \]

2) Label the coefficients and the constant:
   \[
   \begin{array}{ccc}
   a & b & c \\
   1 & -4 & -3 \\
   \end{array}
   \]

   We see that \( a = 1 \), and \( b = -4 \)

3) Place the coefficients into the vertex formula: \( x = -\frac{b}{2a} \)

   \[
   x = -\frac{-4}{2(1)}
   \]

   \[
   x = \frac{4}{2}
   \]

   \[
   x = 2
   \]

   The \( x \)-coordinate of the vertex is 2.

4) To find the \( y \)-coordinate, first, change \( f(x) \) to \( y \).

   \[ f(x) = x^2 - 4x - 3 \rightarrow y = x^2 - 4x - 3 \]

5) Then, plug the \( x \) value into the original function and solve for \( y \).

   \[ y = (2)^2 - 4(2) - 3 \]

   \[ y = 4 - 8 - 3 \]

   \[ y = -7 \]

   The \( y \)-coordinate is -7, putting the vertex at \((2, -7)\).
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Complete the Square Method

You may have used the Complete the Square method to solve for x. Sometimes we use complete the square to find the vertex of a parabola. We want to put the function into the vertex form of a quadratic function: \( y = a(x - h)^2 + k \). In this example, the leading coefficient is a 1, as in \( 1x^2 - 10x + 21 \):

\[
f(x) = x^2 - 10x + 21
\]

We have a trinomial, a polynomial with 3 terms. We can change the \( f(x) \) to \( y \).

\[
y = x^2 - 10x + 21
\]

Divide the coefficient of the middle term by 2, and square it:

\[
- \frac{10}{2} = -5
\]

\[
(-5)^2 = 25
\]

Add this quantity to each side of the equation.

\[
y + 25 = x^2 - 10x + 21 + 25
\]

Group the first two terms with the newly added quantity in a set of parentheses.

\[
y + 25 = (x^2 - 10x + 25) + 21
\]

Factor this trinomial grouping into its perfect square factors.

\[
y + 25 = (x - 5)(x - 5) + 21
\]

Simplify the expression, writing as a squared term with an exponent.

\[
y + 25 = (x - 5)^2 + 21
\]

Subtract (or add) to get \( y \) by itself.

\[
y = (x - 5)^2 + 21 - 25
\]

Simplify:

\[
y = (x - 5)^2 - 4
\]

The function is now in the vertex form of a quadratic function: \( y = a(x - h)^2 + k \)

In this form you can determine the vertex \( (h, k) \), where \( h \) is the x-value of the vertex, and \( k \) is the y-value of the vertex. (The value \( a \) is the coefficient of the first term. In our example, \( a = 1 \).)

Our vertex is \((5, -4)\). Watch the signs of the x- and y-values, so that you do not change the signs in the vertex formula.

When \( a \neq 1 \), the trick is to “factor out” the \( a \), and be careful to add the correct quantity to each side of the equation. Try this example:

\[
y = 3x^2 + 24x - 17
\]

\[
y = 3(x^2 + 8x) - 17
\]

\[
y + 3(16) = 3(x^2 + 8x + 16) - 17
\]

\[
y = 3(x + 4)(x + 4) - 17 - 3(16)
\]

\[
y = 3(x + 4)^2 - 65
\]

\[
h = -4, \; k = -65
\]

The vertex is \((-4, -65)\).