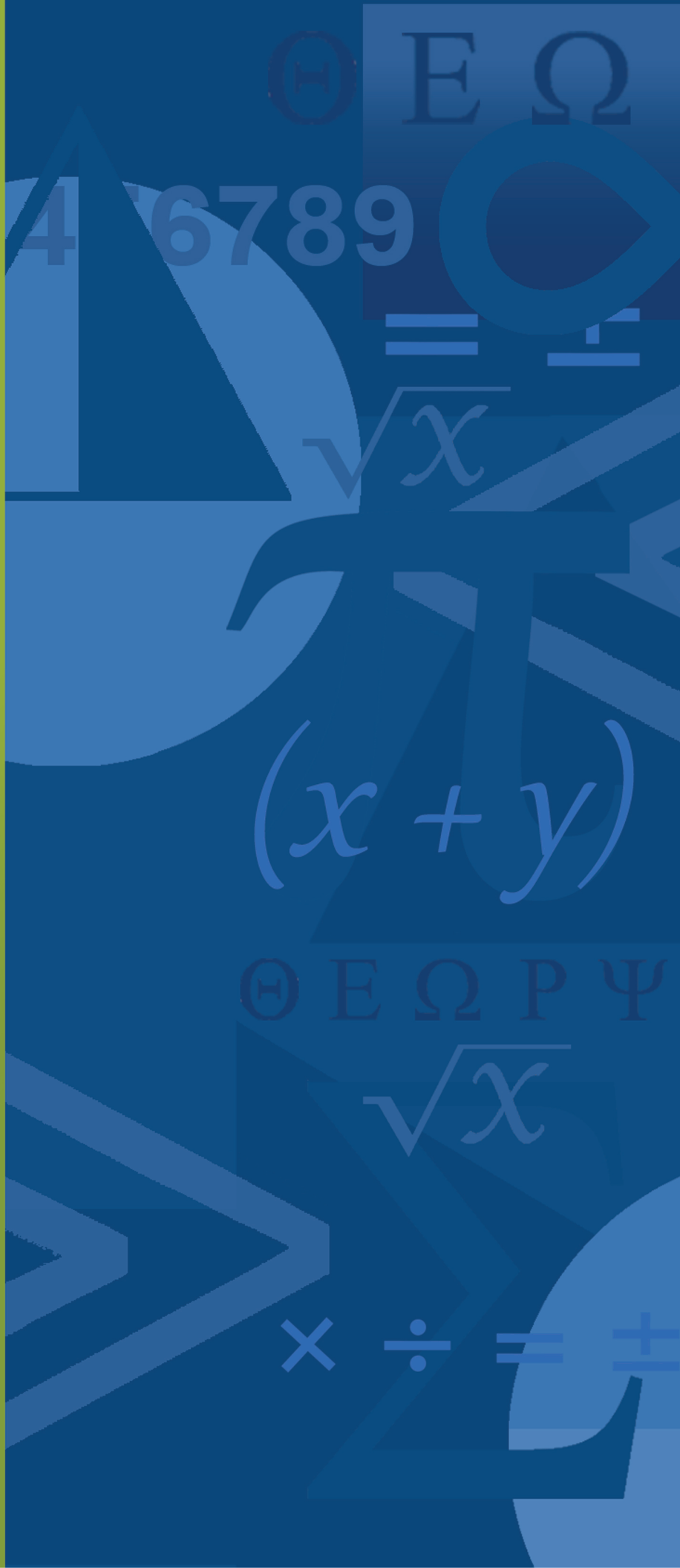
Custom Edition
School of Mathematics

MAT0018C

PREALGEBRA

Concepts and Skills

First Edition



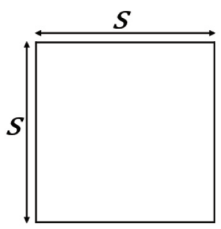
GEOMETRY

SHAPES AND SOLIDS

SQUARE

$P = 4s$

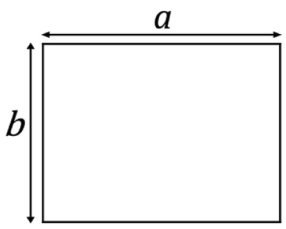
$A = s^2$



RECTANGLE

$P = 2a + 2b$

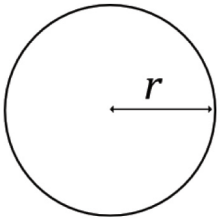
$A = ab$



CIRCLE

$P = 2\pi r$

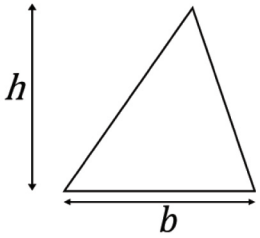
$A = \pi r^2$



TRIANGLE

$P = a + b + c$

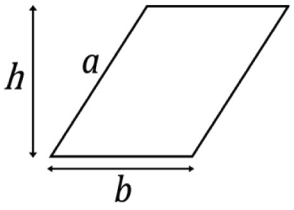
$A = \frac{1}{2}bh$



PARALLELOGRAM

$P = 2a + 2b$

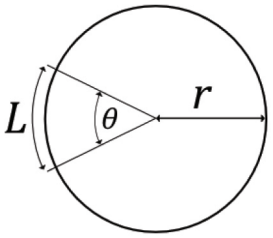
$A = bh$



CIRCULAR SECTOR

$L = \pi r \frac{\theta}{180°}$

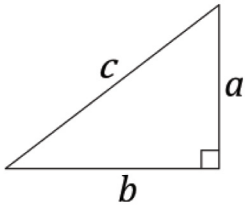
$A = \pi r^2 \frac{\theta}{360°}$



PYTHAGOREAN THEOREM

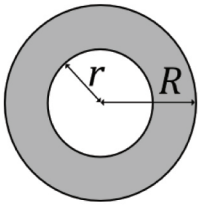
$a^2 + b^2 = c^2$

$c = \sqrt{a^2 + b^2}$



CIRCULAR RING

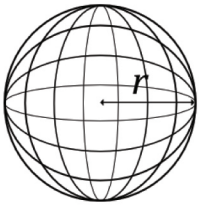
$A = \pi(R^2 - r^2)$



SPHERE

$S = 4\pi r^2$

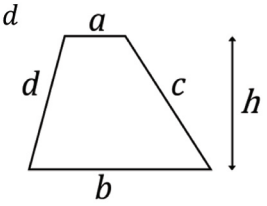
$V = \frac{4\pi r^3}{3}$



TRAPEZOID

$P = a + b + c + d$

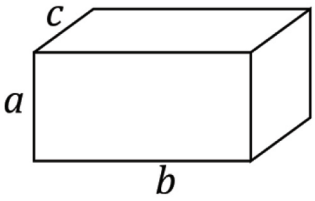
$A = \frac{1}{2}h(a + b)$



RECTANGULAR BOX

$A = 2ab + 2ac + 2bc$

$V = abc$

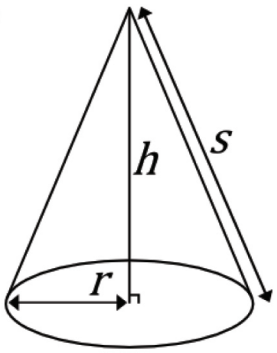


RIGHT CIRCULAR CONE

$A = \pi r^2 + \pi rs$

$s = \sqrt{r^2 + h^2}$

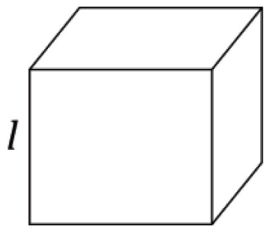
$V = \frac{1}{3}\pi r^2 h$



CUBE

$A = 6l^2$

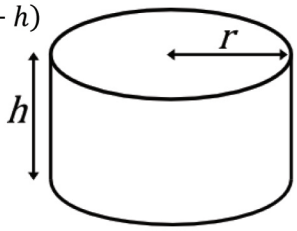
$V = l^3$



CYLINDER

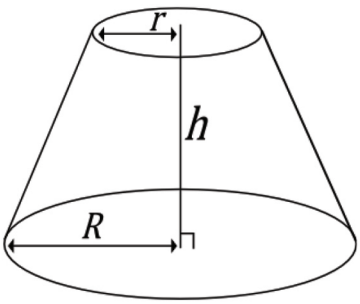
$A = 2\pi r(r + h)$

$V = \pi r^2 h$



FRUSTUM OF A CONE

$V = \frac{1}{3}\pi h(r^2 + rR + R^2)$



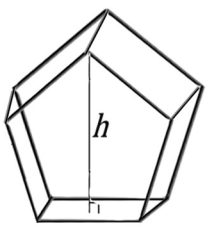
OTHER PRISMS

$A = Ph + 2B$

$V = Bh$

B = area of base

P = perimeter of base



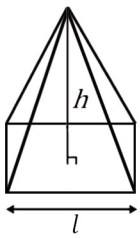
PYRAMID

$A = \frac{1}{2}Pl + B$

$V = \frac{1}{3}Bh$

B = area of base

P = perimeter of base



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PREALGEBRA

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Preface

Pre-Algebra is the overview of the fundamental ideas of Algebra. No matter how long ago you attended high school, you have previous knowledge of the concepts taught in this course. You have made a smart decision to take Pre-Algebra. This course will remind you of the basics of math you might have forgotten and introduce you to the beginning of Algebra concepts. Whether you will or will not need an excess of math courses for your future degree, Pre-Algebra will be the starting point for success in all of your future math courses.

The textbook gives you an excellent opportunity for the study and practice of your basic math skills. It includes detailed examples with instruction and practice problems to help you solidify your knowledge of each concept. This course is designed so teaching and learning all happen in the same class period. You will be able to begin homework and get questions answered about your homework during class.

Although technology seems to be expanding into every detail of our lives, this math class requires a simple calculator to complete some computations, and a computer (provided in the Academic Support Center) will be used to complete modules that show how what you are learning can be used every day in your life.

With this textbook and classroom instruction, you will be well on your way to becoming a successful math student.

Pedagogy

Our pedagogy is simple and powerful. We provide students with immediate feedback from the instructor and plenty of practice with those concepts that will be essential and integral to their future work in mathematics.

Philosophy

We believe the key to learning mathematics, at any level, is class attendance and active participation. That is,"Time on Task + Attendance = Success." When students are actively involved in the process, learning takes place.

With daily practice, immediate feedback, and regular review and reinforcement of previously learned skills, students strengthen their foundational skills while mastering new material. This reinforces the sequential nature of mathematics, leading to better understanding, success in the classroom, and students who are more confident in their quantitative reasoning skills.

At Daytona State College, we strive to provide a positive, comfortable environment that enhances the learning process and is conducive to student success.

Acknowledgments

We wish to thank the many mathematics instructors at Daytona State College who have given their time and effort to make this book a valuable tool for our students.

There were many different aspects of creating this textbook and no one could look back on a project of this magnitude without gratefully acknowledging the efforts of others who contributed so much of their time and the benefit of their knowledge. We wish to thank each of the instructors for contributing to the following different features of the book.

Explanation of Contents: Margaret Overbey, Paul Capps, Gabrielle Booth, Erika Blanken, Ethan Repyneck, Barry Gibson, Andrea Gibson, Marc Campbell, Rachel Repyneck, Heather Bergmann, and Jeffrey Zahnen

Creation of Chapter Exercises: Jim Bishop, Paul Grau, Kathleen Korbich, Benjamin Landon, Judson Stryker, Ethan Repyneck, Lei Wang, and Jeff Zahnen

Text Review: Dana Davidson

Solution Set: Deja Bailey, Gregg Baker, Oliver Vandagriff, Michael Dodge, Trudy White, Valentin Sotomayor, Clyde Hopple, and Marc Campbell

Typed and Formatted: Karthikeya (Sameer) Mamillapalle and Erik Sweet

Mathematical Accuracy: Marc Campbell and Erik Sweet

Other Contributions: Robert Dwarika

In closing, it should be noted that no participants received a stipend or royalty for their participation in this project.

A Final Word

In the beginning of this project, we hoped this book would make a difference in the manner in which algebra is presented and learned as we move forward. Based on the numerous positive comments we have received at the end of this project, we (as a faculty) feel we are definitely on the right path to making a difference in the lives of our students.

Reviewing Mathematical Concepts

At the end of some examples and at the end of the chapters, we have included videos to provide students with several different approaches to help them review and reinforce the ideas and concepts they have to learn. Look for the video symbol and click .

An Introduction to Scientific Calculators

The purpose of this introduction is to explain how to operate a scientific calculator for the problems you will encounter within this textbook. The instructions included here apply to calculators made by Texas Instruments, Sharp, Casio, and Radio Shack. This introduction is not meant to be an exhaustive explanation of all of the capabilities of your scientific calculator but rather a brief overview. For detailed instructions on your calculator, you should refer to the owner's manual.

Arithmetic Operations

To perform an arithmetic operation (adding, subtracting, multiplying, or dividing two numbers), simply enter the first number, press the operation key $\left(\boxed{+}, \boxed{-}, \boxed{\times}, \boxed{\div}\right)$, enter the second number, and then press the $\boxed{=}$ key. On some calculators, the $\boxed{=}$ key is $\boxed{\text{Enter}}$.

For example, divide 24 by 3 using the following keystrokes:

$$\boxed{24}\ \boxed{\div}\ \boxed{3}\ \boxed{=}$$

The display should read $\boxed{8}$.

Change Sign Key

The key marked [±] changes the sign of the display. This is useful in entering negative numbers. For example, enter −5, using the following keystrokes:

[5] [±]

The display should read [−5].

Memory Key

Scientific calculators have the ability to hold numbers in memory for later use. The memory key varies from model to model. Two common memory keys are [M] and [STO].

The [M+] and [M−] keys allows you to add or subtract from the value that is currently being held in memory.

The memory recall key, [MR], [RM], or [RCL], gives you the ability to retrieve the value stored in memory.

If you wish to store the number 8 in memory, use the following keystrokes:

[8] [M]

If your calculator has the [STO] key, use that instead of [M].

To retrieve the stored number from memory, press the key for memory recall on your calculator.

Clearing/Clear Entry Keys

The keys labeled as [C] or [CE] allow you to clear the display or clear the last entry entered into the display.

On some calculators, pressing [C] once will clear the last entry, while pressing [C] twice will clear the entire operation in progress.

Second Function Key

This key is used together with another key to access a function that is printed above an operation key and not on a key itself. On most calculators, the second function key is labeled [2nd].

Suppose you wish to square a number and the squaring function, x^2, is printed above another key. You would have to press [2nd] before the squaring function can be accessed.

Square Root Key

The square root of a number can be found by pressing the $\sqrt{x}$ key.

To find the square root of 49, use the following key strokes:

4 9 $\sqrt{x}$. The display should read 7.

The square root of 2 is an Irrational Number that the calculator can approximate. To find the approximation press

2 $\sqrt{x}$. The display should give the approximation 1.41421436.

The Squaring Key

The squaring key, x^2, allows you to square the entry in the display. To square 21, use the following keystrokes:

2 1 x^2. The display should read 441.

Note that the squaring key and the square root key are typically located on the same key, where one of the functions is the "second" function and can be accessed by first pressing 2nd. For example, if your calculator has the square root as the second function, you would use the following keystrokes to find the square root of 8:

8 2nd x^2. The display should give the approximation 2.82842712.

Reciprocal Key

The reciprocal key is typically labeled as either $1/x$ or x^{-1}. To find the reciprocal of 4, use the following keystrokes,

4 $1/x$. The display should read 0.25.

The Inverse Key

Some calculators have a key marked [INV]. This is the inverse key. The use of the inverse key varies from calculator to calculator, so read your owner's manual for a detailed explanation of this key. However, keep in mind that an inverse operation undoes the actions of some other operation. The [INV] key is used to access an inverse operation.

Exponential Key

The key marked as either [x^y] or [y^x] allows you to raise an entry to a desired power. For example, if you want to raise the number 2 to the power 5, use the following keystrokes.

[2] [x^y] [5] [=]. The display should read [32].

Root Key

This is the key marked as either [$\sqrt[x]{y}$] or [$\sqrt[y]{x}$]. If you do not have this key on your calculator, you can access the root function by using the [INV] key.

Suppose that you wish to find the fifth root of the number 1024. Use the following keystrokes.

[1] [0] [2] [4] [$\sqrt[x]{y}$] [5] [=].

The display should read [4].

If your calculator does not have either [$\sqrt[x]{y}$] or [$\sqrt[y]{x}$], use

[1] [0] [2] [4] [INV] [x^y] [5].

The inverse operation of raising something to the nth power is taking the nth root.

Pi Key

An important irrational number that you will use often in mathematics is the number π. The value of π is stored in your calculator and can be accessed using the [π] key.

Methods of Display

When decimal approximations are shown on scientific calculators, they are either rounded or truncated. To determine which method your calculator uses, evaluate $1/18$ by using the following keystrokes:

1	÷	1	8	=

If your display shows | 0.0555555 |, your calculator truncates the display. If your display reads | 0.0555556 |, it rounds the display.

When very small or very large numbers need to be displayed, scientific calculators often use Scientific Notation.

For example, if you multiply $23,456$ and $17,045$, your calculator may display

| 3.988576 e +9 | or | 3.988576 9 |.

These both mean that you should multiply the number 3.988576 by 10^9.

If your calculator displays

| 2.54 e -5 | or | 2.54 -5 |,

it means that you should multiply 2.54 by 10^{-5}.

WHOLE NUMBERS AND THEIR APPLICATIONS

CHAPTER 1

CHAPTER OBJECTIVES

1.1 Addition and Subtraction of Whole Numbers and Calculating Perimeter

1.2 Rounding, Multiplication and Division of Whole Numbers and Calculating Area

1.3 Order of Operations

1.4 Problem Solving

The Climb

Imagine watching a rock climber. Notice how carefully he examines the route ahead to make choices for the next move. He has four contacts with the rock - with his two hands and his two feet.

A basic rule for rock climbing is to move only one contact at a time. With one hand or one foot, he experiments with the next move that will take him up and keep him secure. **The important issues for him are controlling the situation and continuing to move along a route with positive options**.

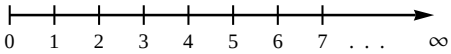
Be like the rock climber. Take control, keep your solid contacts with the ground, and move in directions with positive options. Every once in a while, pause and take a look back to see how far you have come.

Then this will be a safe journey for you because you will choose the rocks that you are willing to climb and you will take charge of each moment. Your journey will progress one move at a time. Your focus will be on the “here and now” as you choose from your currently available options.

1.1 Addition and Subtraction of Whole Numbers and Calculating Perimeter

In this section we investigate the mathematical operations of addition and subtraction on the set of Whole Numbers. The set of **Whole Numbers**, denoted as $\mathbb{W}$, is sometimes referred to as the "counting numbers" as the set contains the numbers $0, 1, 2, 3, \ldots$.

When learning how to add and subtract whole numbers, it can be useful to visualize the numbers on a line. That is,



can be used the represent the whole numbers starting at 0 and increasing forever. The arrow going off to the right indicates that the numbers keep getting bigger and the symbol "∞", the infinity symbol, indicates that the numbers go on forever.

When adding whole numbers it is also useful to know that each digit of a whole number has a place value. For convenience in reading large numbers, commas are used to separate groups of three digits counting from right to left. The value of each digit in the whole number is a product of the digit and its place value.

As an example, consider the number $987,654,321$. The chart provided below illustrates each digit's place value. That is, the digit 5 in the whole number is in the Ten Thousands place. This means that its value to the whole number is five, $10,000$'s or $50,000$.

Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
9	8	7,	6	5	4,	3	2	1

We begin by adding "small" whole numbers using a number line and move into adding "large" whole numbers using the place value of each digit in the whole number. Adding "large" whole numbers depends on the ability to write whole numbers in expanded form.

1.1.1 Addition of Whole Numbers

In statements using addition, the numbers that are being added are called the **addends** and the result of the addition is called the **sum**. As an example, in the statement $6+7=13$, the numbers 6 and 7 are called the addends and the sum is 13.

Some terminology/phrases used to indicate addition are *plus*, *total*, *increased by*, *added to*, and *more than*. It is important to keep in mind that all of these indicate that we should add two numbers or quantities. As an example, the addition of 8 and 6, $8+6$, can be written in a few different ways. English phrases representing $8+6$ are

- 8 plus 6
- 8 increased by 6
- 6 added to 8
- The total of 8 and 6
- 6 more than 8
- Add 8 and 6

Each statement represents $8+6=14$.

Notice that the order of the numbers in symbolic form is the same as the English form except when we use the phrase "more than" or "added to." In these cases the number following the phrase is considered the starting value, and the first number is added to the starting value.

On a number line, addition implies "move to the right." The following examples illustrate this.

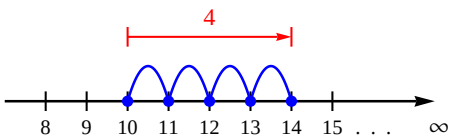
EXAMPLE 1 Write the statement "10 increased by 4" in symbolic form and then simplify.

SOLUTION: The symbolic form of this statement would be $10+4$ and it simplifies to 14.

That is,

$$10+4=14.$$

To calculate this using a number line, we start at the number 10 and move to the **right** 4 units. Since we end up at the number 14, $10+4=14$.



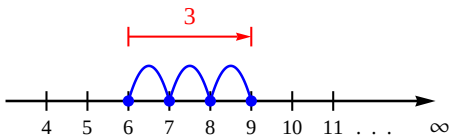
EXAMPLE 2 Write the statement "3 added to 6" in symbolic form and then simplify.

SOLUTION: The symbolic form of this statement would be $6+3$ and it simplifies to 9.

That is

$$6+3=9.$$

To calculate this using a number line, we start at the number 6 and move to the **right** 3 units. Since we end up at the number 9, $6+3=9$.



In many applications you will be asked to add more than two numbers at a time. To accomplish this keep the following helpful hint in mind.

Helpful Hint: Adding More Than Two Numbers

In order to calculate the sum of three or more numbers we can begin on the left by adding two numbers at a time. For example,

$$5+2+6+1=(5+2)+6+1=7+6+1=(7+6)+1=13+1=14.$$

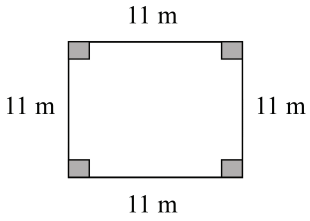
EXAMPLE 3 Write the statement "The total of 3, 6, and 7" in symbolic form and then simplify.

SOLUTION: The symbolic form of this statement would be $3+6+7$ and it simplifies to 16. That is

$$3+6+7=(3+6)+7=9+7=16.$$

The **Perimeter** of an object is found by adding the lengths of all of its sides. The units used in perimeter are the same as the units used to express the lengths of the sides of the object. When finding perimeter, make sure that all the addends are expressed using the same units of measurement.

EXAMPLE 4 Find the perimeter of the given shape.



SOLUTION: To find the perimeter, P, we need to sum the lengths of the sides of the object. Therefore,

$$P=11\,\text{m}+11\,\text{m}+11\,\text{m}+11\,\text{m}=44\,\text{m}.$$

The perimeter of the object is 44 m.

Expanded Form of Whole Numbers

EXAMPLE 5 Write the number $2,345$ in expanded form.

SOLUTION: Writing a whole number in expanded form is the same as recognizing the place value of each digit in the whole number.

In this example, the digit 5 is in the ones place, the digit 4 is in the tens place, 3 is in the hundreds place, and 2 is in the thousands place. So, the expanded form of $2,345$ is

$$2,000+300+40+5.$$

EXAMPLE 6 Write the number $5,209$ in expanded form.

SOLUTION: The digit in the thousands place is 5, the digit in the hundreds place is 2, zero is in the tens place, and 9 is in the ones place. So

$$5,209 = 5,000+200+9.$$

Addition of Large Whole Numbers in Expanded Form

Using a number line to add small whole numbers can be useful but it becomes difficult when adding numbers such as 409 and 187. To add larger whole numbers we write each number in expanded form and then add the digits in corresponding place values.

EXAMPLE 7 Add 34 and 27.

SOLUTION: Write each number in expanded form.

$$34 = 30+4, \quad \text{and} \quad 27 = 20+7.$$

Begin by adding the digits in the ones place.

$$(30+4)+(20+7) = 30+20+11$$

Adding the digits in the ones place gave the number $11 = 10+1$. We can add this additional ten to the existing tens.

$$30+20+11=30+20+10+1.$$

Now add the digits in the tens place.

$$30+20+10+1=60+1$$

The sum of 34 and 27 is then

$$34+27=60+1=61.$$

EXAMPLE 8 Add 503 and 625.

SOLUTION: Writing each number in expanded form gives

$$503=500+3, \quad \text{and} \quad 625=600+20+5.$$

Beginning in the ones place and moving to tens and then the hundreds place we get the following.

$$500+3+600+20+5=500+600+20+8$$

$$=500+600+20+8$$

$$=500+600+20+8$$

$$=1,100+20+8$$

$$=1,100+20+8=1,128.$$

Addition of Large Whole Numbers Vertically

The technique of writing each number of a sum in expanded form and then adding digits in corresponding place values can be streamlined a bit by adding the numbers vertically.

EXAMPLE 9 Add 865 and 98.

SOLUTION: Write the larger addend above the smaller by lining up the place values of each number.

$$\begin{array}{r} 8\ 6\ 5 \\ +\quad 9\ 8 \\ \hline \end{array}$$

Adding the digits in the ones place gives $13 = 10 + 3$. So,

$$\begin{array}{r} 1 \\ 8\ 6\ 5 \\ +\quad 9\ 8 \\ \hline 3 \end{array}$$

Adding the numbers in the tens place gives $10 + 60 + 90 = 160$. So,

$$\begin{array}{r} 1\ 1 \\ 8\ 6\ 5 \\ +\quad 9\ 8 \\ \hline 6\ 3 \end{array}$$

Finally, adding the numbers in the hundreds place gives $100 + 800 = 900$. So we get

$$\begin{array}{r} 1\ 1 \\ 8\ 6\ 5 \\ +\quad 9\ 8 \\ \hline 9\ 6\ 3 \end{array}$$

Therefore, $865 + 98 = 963$.

EXAMPLE 10 Add $3,098$ and $1,370$.

SOLUTION: Write the larger addend above the smaller by lining up the place values of each number.

$$\begin{array}{r} 3\ 0\ 9\ 8 \\ +\ 1\ 3\ 7\ 0 \\ \hline \end{array}$$

Beginning in the ones place and moving to the left we get the following.

$$\begin{array}{r} {\scriptstyle 1} \\ 3\ 0\ 9\ 8 \\ +\ 1\ 3\ 7\ 0 \\ \hline 4\ 4\ 6\ 8 \end{array}$$

Therefore, $3,098 + 1,370 = 4,468$.

1.1.2 Subtraction of Whole Numbers

Just as with addition, we will begin by subtracting "small" whole numbers using a number line and then discuss the technique of subtracting "larger" whole numbers.

The numbers involved in subtraction are referred to as the **minuend** and the **subtrahend**. The subtrahend is the number that is subtracted from the minuend. For example, in the statement $8-3$, the 3 is being subtracted from the 8, so we call 3 the subtrahend and we call 8 the minuend. The **difference** is the result of subtraction. So the difference of 8 and 3, namely $8-3$, is 5.

Similar to addition, we can use different terminology to describe subtraction. As an example, the symbolic statement $18-6$ can be written in a few different ways.

- The difference of 18 and 6
- 18 minus 6
- 18 less 6
- 18 decreased by 6
- 6 less than 18
- 6 subtracted from 18

All of these statements are equivalent in that they all mean $18-6$ and that they all simplify to 12.

Recall that on a number line, the addition of a number indicated that we should move to the right. Here, on a number line subtraction implies "move to the left." The following examples illustrate this.

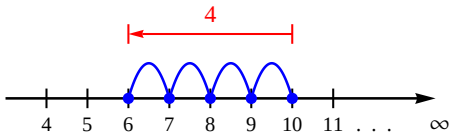
EXAMPLE 1 Write the statement "10 decreased by 4" in symbolic form and then simplify.

SOLUTION: The symbolic form of the statement would be $10-4$ and it simplifies to 6.

That is,

$$10-4=6.$$

To calculate this using a number line, we start at the number 10 and move to the **left** 4 units.



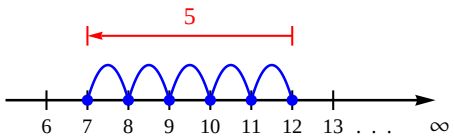
EXAMPLE 2 Write the statement "5 subtracted from 12" in symbolic form and then simplify.

SOLUTION: The symbolic form of the statement would be $12-5$ and it simplifies to 7.

That is,

$$12-5=7.$$

To calculate this using a number line, we start at the number 12 and move to the **left** 5 units.



EXAMPLE 3 Write the statement "The difference of 9 and 5" in symbolic form and then simplify.

SOLUTION: The symbolic form of the statement would be $9-5$ and it simplifies to 4.

That is,

$$9-5=4.$$

EXAMPLE 4 Write the statement "11 less than 20" in symbolic form and then simplify.

SOLUTION: The symbolic form of the statement would be $20-11$ and it simplifies to 9.

That is,

$$20-11=9.$$

Helpful Hint: Subtracting More Than Two Numbers

In order to calculate the difference of three or more numbers we can begin on the left by subtracting two integers at a time. For example,

$$15-4-6-2=(15-4)-6-2=11-6-2=(11-6)-2=5-2=3.$$

EXAMPLE 5 Evaluate the expression $10-2-5$

SOLUTION: To evaluate this expression, start at the left by subtracting two integers at a time.

That is,

$$10-2-5=(10-2)-5=8-5=3$$

EXAMPLE 6 Evaluate the expression $22-3-12-1$

SOLUTION: To evaluate this expression, start at the left by subtracting two integers at a time.

$$22-3-12-1=(22-3)-12-1=19-12-1=(19-12)-1=7-1=6$$

EXAMPLE 7 The original price of a jacket was \$39. Maric paid \$12 less than the original price. What did Maric pay for the jacket?

SOLUTION: In the statement "\$12 dollars less than the original price," replace the "original price" with the actual numerical price so that we get "\$12 dollars less than \$39." Symbolically this statement is written as

$$\$39 - \$12.$$

Because $\$39 - \$12 = \$27$, Maric paid \$27 for the jacket.

Subtraction of Large Whole Numbers in Expanded Form

EXAMPLE 8 Subtract 45 from 68.

SOLUTION: Write the minuend and subtrahend in expanded form.

$$45 = 40 + 5, \quad \text{and} \quad 68 = 60 + 8.$$

To calculate $68 - 45$, subtract the numbers in the ones place and then subtract the numbers in the tens place. In the ones place, $8 - 5 = 3$ and in the tens place, $60 - 40 = 20$. So,

$$68 - 45 = 20 + 3 = 23.$$

EXAMPLE 9 Subtract 187 from 363.

SOLUTION: Write the minuend and subtrahend in expanded form.

$$300 + 60 + 3 \quad \text{and} \quad 187 = 100 + 80 + 7.$$

This means the subtraction $363-187$ can be written as

$$363-187=(300+60+3)-(100+80+7).$$

Beginning with the ones place we have $3-7$ which is a problem. To overcome this problem we can **borrow** a ten from the 363. That is,

$$363=300+60+3=300+50+10+3=300+50+13.$$

So now we have $363-187$ written as

$$(300+50+13)-(100+80+7)$$

so that in the ones place we have $13-7=6$.
In the tens place we would have to calculate $50-80$ which is also a problem. To overcome this we can again **borrow** but this time we borrow a hundred from the 363. That is

$$300+50+13=200+100+50+13=200+150+13.$$

So now we have $363-187$ written as

$$(200+150+13)-(100+80+7).$$

Performing the subtraction in the ones place we get

$$13-7=6.$$

Performing the subtraction in the tens place we get

$$150-80=70.$$

Subtracting the hundreds gives

$$200 - 100 = 100.$$

Therefore, $363 - 187 = 100 + 70 + 6 = 176$.

Subtraction of Large Whole Numbers Vertically

Just as with addition, subtraction can be completed vertically as shown here.

EXAMPLE 10 Simplify $458 - 197$.

SOLUTION: Write the larger number above the smaller number by lining up the corresponding place values.

$$\begin{array}{r} 4\ 5\ 8 \\ -\ 1\ 9\ 7 \\ \hline \end{array}$$

In the ones place we have $8 - 7 = 1$. So we get

$$\begin{array}{r} 4\ 5\ 8 \\ -\ 1\ 9\ 7 \\ \hline \quad\quad 1 \end{array}$$

In the tens place we have $50 - 90$ so we need to borrow a hundred from 400 to get $150 - 90$. With the vertical set-up we write the following.

$$\begin{array}{r} \quad 3\ 15 \\ \cancel{4}\ \cancel{5}\ 8 \\ -\ 1\ 9\ 7 \\ \hline \quad\quad 6\ 1 \end{array}$$

Complete the subtraction by subtracting the numbers in the hundreds place.

$$\begin{array}{r} {\scriptstyle 3\ 15} \\ 4\ \not{5}\ 8 \\ -\ 1\ 9\ 7 \\ \hline 2\ 6\ 1 \end{array}$$

Therefore, $458 - 197 = 261$.

EXAMPLE 11 Perform the subtraction $9,885 - 7,877$.

SOLUTION: Write the larger number above the smaller number by lining up the corresponding place values.

$$\begin{array}{r} 9\ 8\ 8\ 5 \\ -\ 7\ 8\ 7\ 7 \\ \hline \end{array}$$

$$\begin{array}{r} {\scriptstyle 7\ 15} \\ 9\ 8\ \not{8}\ \not{5} \\ -\ 7\ 8\ 7\ 7 \\ \hline 2\ 0\ 0\ 8 \end{array}$$

1.1.3 Order of Operations: Addition and Subtraction of Whole Numbers

Order of Operations: Addition and Subtraction of Whole Numbers
Addition and Subtraction of Whole Numbers
Perform addition and subtraction as they occur from left to right.

The helpful hints provided within the section said that in order to add, or subtract, three or more numbers, we should begin at the left of the expression, group two numbers at a time and move to the right through the expression. We can use this idea to combine the operations of addition and subtraction as illustrated in the next examples.

EXAMPLE 1 Evaluate the expression $10+5-9+1-3$

SOLUTION: Using Order of Operations, perform addition and subtraction as they appear from left to right.

$$10+5-9+1-3=(10+5)-9+1-3$$

$$=15-9+1-3$$

$$=(15-9)+1-3$$

$$=6+1-3$$

$$=(6+1)-3$$

$$=7-3$$

$$=4.$$

EXAMPLE 2 Evaluate the expression $31-19-11+13$

SOLUTION: Using Order of Operations we get

$$31-19-11+13=(31-19)-11+13$$

$$=12-11+13$$

$$=(12-11)+13$$

$$=1+13$$

$$=14$$

Section 1.1 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 3, identify the addends and the sum.

1) $3+9=12$ 2) $19+21=40$ 3) $12+22=34$

For problems 4 - 9, first translate the English phrase into an equivalent mathematical statement and then simplify.

4) The sum of 6 and 4 5) 5 plus 7 6) 16 added to 21

7) 8 more than 6 8) The total of 4 and 20 9) 200 increased by 50

For problems 10 - 12, perform the addition by adding the numbers vertically.

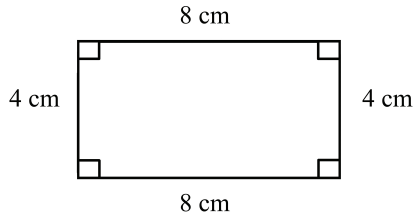
10) $467+250$ 11) $5,872+3,903$ 12) $12,671+462$

For problems 13 and 14, use Order of Operations to evaluate the expressions.

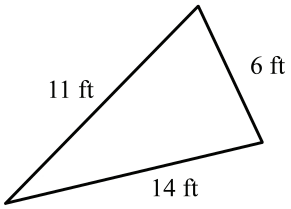
13) $5+3+6+8$ 14) $15+2+9+3+5$

For 15 - 17, find the perimeter of the given object.

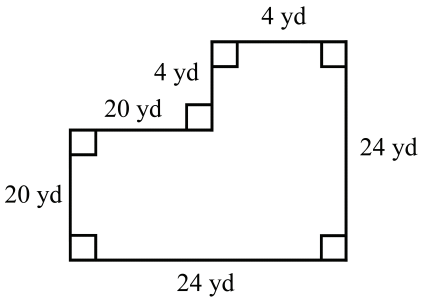
15)



16)



17)



For 18 - 20, identify the minuend, subtrahend, and the difference.

18) $35-16=19$ 19) $105-27=78$ 20)

$$\begin{array}{r} 8 \\ -\ 2 \\ \hline 6 \end{array}$$

For 21 -26, first translate the English phrase into an equivalent mathematical statement then simplify.

21) The difference of 15 and 10.

22) 100 minus 80.

23) 30 less than 50.

24) 20 subtracted from 50.

25) 98 decreased by 70

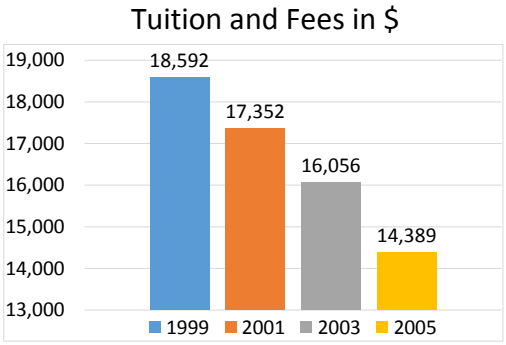
For 26 -28, perform the subtraction by subtracting the numbers vertically.

26) $512 - 342$

27) $2,308 - 1,726$

28) $1,486 - 523$

For 29 - 31, use the information provided in the graph



29) What is the difference in the cost tuition between 2001 and 2005?

30) Between which two 2-year periods did the greatest decrease in the cost of tuition occur?

31) How much did the cost of tuition decrease between the years 1999 and 2005?

For problems 32 - 25, use Order of Operations to evaluate the expression.

32) $9 - 3 - 4$

33) $12 - 4 + 8 + 2$

34) $5 - 3 + 8 + 4 - 6$

35) $12 - 4 - 1 - 5$

Chapter 1
Section 1.1 – Addition and Subtraction of Whole Numbers and Calculating Perimeter

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

1) Addends: _____

   Sum: _____

2) Addends: _____

   Sum: _____

3) Addends: _____

   Sum: _____

4) _____

5) _____

6) _____

7) _____

8) _____

9) _____

10) _____

11) _____

12) _____

13) _____

14) _____

15) _____

16) _____

17) _____

18) Minuend: _____

   Subtrahend: _____

   Difference: _____

19) Minuend: _____

   Subtrahend: _____

   Difference: _____

20) Minuend:_____

   Subtrahend: _____

   Difference: _____

21) _____

22) _____

23) _____

24) _____

25) _____

26) _____

27) _____

28) _____

29) _____

30) _____

31) _____

32) _____

33) _____

34) _____

35) _____

1.2 Rounding, Multiplication and Division of Whole Numbers and Calculating Area

Recall from Section 1.1, each digit of a whole number has a place value. For convenience in reading large numbers, commas are used to separate groups of three digits counting from right to left. The value of each digit in the whole number is a product of the digit and its place value.

As an example, we considered the number $987,654,321$. The chart provided below illustrates each digit's place value. That is, the digit 5 in the whole number is in the Ten Thousands place. This means that its value to the whole number is five, $10,000$'s or $50,000$.

Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
9	8	7,	6	5	4,	3	2	1

1.2.1 Rounding Whole Numbers

Rounding Whole Numbers

Identify the location of the requested place value to which you want to round.

- If the digit to the right of the requested place value is between 0 and 4, replace that digit and all digits to the right of it with a zero.
- If the digit to the right of the requested place value is between 5 and 9, add one to the digit in the requested place value and replace all digits to the right of that place value with a zero.

EXAMPLE 1 Round $37,268$ to the nearest thousand.

SOLUTION: The digit in the thousands place is 7. The digit to its right is 2. Because the 2 is between 0 and 4, we replace the 2 and all of the digits to the right of it with 0.

$37,268$ rounded to the nearest thousand is $37,000$.

EXAMPLE 2 Round $37,268$ to the nearest hundred.

SOLUTION: The digit in the hundreds place is 2. The digit to its right is 6. Because the 6 is between 5 and 9, we add one to the 2 and replace all of the digits to the right of it with a 0.

$37,268$ rounded to the nearest hundred is $37,300$.

EXAMPLE 3 Round $29,875$ to the nearest ten-thousand.

SOLUTION: The digit in the ten-thousandth place is a 2. The digit to its right is 9. Because 9 is between 5 and 9, we add one to the 2 and replace all of the digits to the right of it with a 0.

$29,875$ rounded to the nearest ten-thousand is $30,000$.

1.2.2 Multiplication of Whole Numbers: Repeated Addition

Multiplication by Zero

Thinking of multiplication as repeated addition implies that any non-zero number a times zero must be zero since $a \cdot 0$ can be thought of as a added 0 times. Thus, $a \cdot 0 = 0$.

The operation of **multiplication** is defined as *repeated addition*. The numbers being multiplied are called **factors** and the result of the multiplication is called the **product**.

Just as in addition and subtraction, we can denote the operation of multiplication a couple of ways. For example, symbolically we can write 3 times 5 as

$$3 \times 5 \quad \text{or} \quad 3 \cdot 5 \quad \text{or} \quad 3(5).$$

All three representations mean the exact same thing.

Since multiplication is simply repeated addition, the statement 3×5 means

$$5+5+5 \text{ or } 3+3+3+3+3$$

both of which simplify to 15. That is, the product of 3 and 5 is 15.

The next few examples illustrate some applications that utilize multiplication.

EXAMPLE 1 Mark is taking 4 classes. Each class meets 3 hours per week. How many hours is Mark in class each week?

SOLUTION: To determine the total number of hours Mark is in class each week we would simply add the number of hours he is in each class every week. That is, Mark is in class for

$$3 \text{ hrs } +3 \text{ hrs } +3 \text{ hrs } +3 \text{ hrs } = 12 \text{ hrs.}$$

We could have written this using multiplication rather than addition. That is, Mark is in class for

$$4 \cdot 3 \text{ hrs } = 12 \text{ hrs.}$$

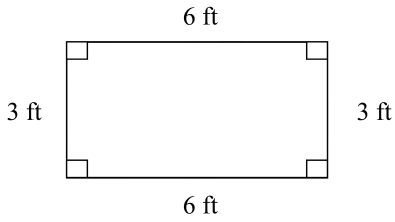
EXAMPLE 2 Jo needs to buy 8 cases of drinks for a party. Each case costs \$5. How much money does he need to buy the drinks?

SOLUTION: Since each case costs \$5 and he needs 8 cases, Jo will need

$$8 \times \$5 = \$40 \text{ to but drinks for the party.}$$

An application of multiplication is that of finding the **Area of a Rectangle**. The area of a rectangle is found by multiplying its length by its width. The resulting product is expressed in square units.

EXAMPLE 3 Find the area of the rectangle shown below.



SOLUTION: The length of the rectangle is 6 ft and the width is 3 ft. The area of the rectangle is the product of its length and its width.

$$\text{Area} = \text{Length} \times \text{Width} = (6\,\text{ft})(3\,\text{ft}) = 18\,\text{ft}^2.$$

Helpful Hint: Multiplying More Than Two Numbers

In order to calculate the product of three or more numbers we can begin on the left by multiplying two numbers at a time. For example,

$$3 \cdot 2 \cdot 4 \cdot 2 = (3 \cdot 2) \cdot 4 \cdot 2 = 6 \cdot 4 \cdot 2 = (6 \cdot 4) \cdot 2 = 24 \cdot 2 = 48.$$

EXAMPLE 4 Simplify the expression $5(3)(3)$.

SOLUTION: Working from left to right we get

$$5(3)(3) = 5(3)(3) = 15(3) = 45.$$

1.2.3 Natural Number Exponents: Repeated Multiplication

Recall that the set of Whole Numbers, denoted as $\mathbb{W}$, contain the Natural Numbers $1, 2, 3, \ldots$. The set of **Natural Numbers** is denoted as $\mathbb{N}$.

In this section we use Natural Numbers and exponents to denote repeated multiplication.

We are able to use the operation of multiplication to represent repeated addition. That is, the expression $5(4)$ means "add 5 to itself 4 times." We can represent repeated multiplication using notation called **Exponents**.

As an example, the expression $7(7)(7)$ means "multiply 7 by itself 3 times." We can represent this as 7^3, read as 7 to the power 3. In this expression we call the number 7 the "base" and the number 3 is called the "exponent."

Natural Number Exponents

Given any whole number a and any natural number n:

$$a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a \cdot a}_{\text{multiply the base } a, \, n \text{ times}} .$$

We call the number a the **base** and n the **exponent**.

EXAMPLE 1 Simplify the expression 3^3.

SOLUTION: The expression 3^3 means "multiply 3 by itself 3 times." So,

$$3^3 = 3(3)(3) = 3(3)(3) = 9(3) = 27.$$

Power of 1

Any number raised to the power of 1 is itself. That is, $a^1 = a$.

EXAMPLE 2 Simplify the expression 2^5.

SOLUTION: The expression 2^5 means "multiply 2 by itself 5 times." So,

$$2^5 = 2(2)(2)(2)(2).$$

Working from left to right we get $2^5 = 32$.

Caution

2^5 is not the same as $2 \cdot 5$. Notice that $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$ while $2 \cdot 5 = 10$.

One very useful application of Natural Number exponents is in finding powers of 10. Consider the following pattern.

- $10^1 = 10$
- $10^2 = 10 \cdot 10 = 100$
- $10^3 = 10 \cdot 10 \cdot 10 = 1,000$
- $10^4 = 10 \cdot 10 \cdot 10 \cdot 10 = 10,000$

$\vdots$

The pattern shown above implies that for any natural number n, 10^n is simply 1 followed by n zeros.

The implication is that any number multiplied by 10^n is that number followed by n zeros.

EXAMPLE 3 Simplify the expression $6(10,000)$.

SOLUTION: Since $6 \cdot 10,000 = 6 \cdot 10^4$, we get

$$6(10,000) = 60,000$$

which is 6 followed by 4 zeros.

Note that adding $10,000$ to itself 6 times produces the same result. It is simply more efficient to perform the multiplication.

EXAMPLE 4 Simplify the expression $13(1,000)$.

SOLUTION: Since $13 \cdot 1,000 = 13 \cdot 10^3$, we get

$$13(1,000) = 13,000$$

which is 13 followed by 3 zeros.

Note again that while adding $10,000$ to itself 13 times will produces the same result, performing the multiplication is more efficient.

EXAMPLE 5 Simplify the expression $6(500)$.

SOLUTION: Since 500 can be written as $5 \cdot 100 = 5 \cdot 10^2$, we get

$$6 \cdot 500 = 6 \cdot 5 \cdot 10^2 = 30 \cdot 10^2 = 3,000$$

which is 30 followed by 2 zeros.

EXAMPLE 6 Simplify the expression $4(80,000)$.

SOLUTION: Since $80,000 = 8 \cdot 10,000 = 8 \cdot 10^4$, we get

$$4 \cdot 80,000 = 4 \cdot 8 \cdot 10^4 = 32 \cdot 10^4 = 320,000$$

which is 32 followed by 4 zeros.

1.2.4 The Distributive Property and Multiplying Large Whole Numbers

The Distributive Property of Whole Numbers

The Distributive Property states that for any whole numbers a, b, and c

$$a(b+c) = a(b) + a(c), \quad \text{and} \quad (a+b)c = a(c) + b(c).$$

The Distributive Property says that we can multiply the sum of two numbers by multiplying each of the addends and then adding the results. As a simple example consider multiplying $5(4+6)$. We could perform the addition first and then perform the multiplication to get

$$5(4+6) = 5(10) = 50$$

or we can multiply each addend and then perform the addition to get

$$5(4+6) = 5(4)+5(6) = 20+30 = 50.$$

This is not all that interesting when dealing with products such as $4(3)$ or $5(9)$, but it becomes useful when multiplying larger numbers such as $65(367)$.

EXAMPLE 1 Use the Distributive Property to multiply 4 and 67.

SOLUTION: Write 67 in expanded form to get $67 = 60+7$. Then, according to the Distributive Property we have

$$4(67) = 4(60+7) = 4(60)+4(7).$$

Simplifying we get

$$240+28 = 268.$$

Therefore, $4(67) = 268$.

EXAMPLE 2 Use the Distributive Property to multiply 39 and 27.

SOLUTION: Write 39 in expanded form to get $39 = 30+9$. Then, according to the Distributive Property we have

$$27(39) = 27(30+9) = 27(30)+27(9).$$

To calculate the products $27(3)$ and $27(9)$, write 27 in expanded form and apply the Distributive Property.

$$27(30)+27(9) = (20+7)(30)+(20+7)(9).$$

$$= 20(30)+7(30)+20(9)+7(9) = 600+210+180+63$$

$$= 1053.$$

EXAMPLE 3 Use the Distributive Property to multiply 438 and 8.

SOLUTION: Write 438 in expanded form to get

$$438 = 400 + 30 + 8.$$

Then, according to the Distributive Property we have

$$8(438) = 8(400 + 30 + 8) = 8(400) + 8(30) + 8(8).$$

Now since $400 = 4 \cdot 100$,

$$8(400) = 8 \cdot 4 \cdot 100 = 32 \cdot 10^2 = 3200.$$

Likewise,

$$8(30) = 8 \cdot 3 \cdot 10 = 24 \cdot 10 = 240.$$

Putting this together gives

$$8(438) = 8(400) + 8(30) + 8(8) = 3200 + 240 + 64 = 3504.$$

Just like we encountered with addition and subtraction of Whole Numbers, multiplication will be more efficient and straightforward if we perform the operation vertically.

1.2.5 Multiplication of Large Whole Numbers Vertically

The Distributive Property provides a means of calculating the products of larger whole numbers but because of the potential for a large number of calculations it can have the disadvantage of becoming cumbersome. Thankfully there is a method that allows us to keep track of the multiplication in a more concise manner.

EXAMPLE 1 Perform the multiplication $87(92)$.

SOLUTION: Write the larger number above the smaller number by lining up the corresponding place values.

$$\begin{array}{r} 9\ \ 2 \\ \times\ 8\ \ 7 \\ \hline \end{array}$$

Begin by multiplying 92 by the number in the ones place of 87, namely 7. Note that

$$7(92) = 7(90+2) = 7(90)+7(2) = 630+14$$

$$630+10+4 = 640+4.$$

We write this vertically as

$$\begin{array}{r} {\scriptstyle +1} \\ 9\ \ 2 \\ \times\ 8\ \ 7 \\ \hline 6\ \ 4\ \ 4 \end{array}$$

Now multiply 92 by the number in the tens place of 87, namely 80. Note that

$$80(92) = 80(90+2) = 80(90)+80(2) = 7200+160$$

$$= 7200+100+60 = 7300+60$$

$$= 7360.$$

We write this vertically as

$$\begin{array}{r} {\scriptstyle +1} \\ 9\ \ 2 \\ \times\ 8\ \ 7 \\ \hline 6\ \ 4\ \ 4 \\ 7\ 3\ 6\ 0 \end{array}$$

To complete the multiplication we add the two results.

$$\begin{array}{r} {\scriptstyle +1} \\ 92 \\ \times\ 87 \\ \hline 644 \\ +\,7360 \\ \hline 8004 \end{array}$$

EXAMPLE 2 Perform the multiplication $314(203)$.

SOLUTION: Write the larger number above the smaller number by lining up the corresponding place values.

$$\begin{array}{r} 314 \\ \times\ 203 \\ \hline \end{array}$$

Begin by multiplying 314 by the number in the ones place of 203.

$$3(314) = 3(300+10+4) = 900+30+12 = 900+40+2.$$

Writing this vertically we have

$$\begin{array}{r} {\scriptstyle +1} \\ 314 \\ \times\ 203 \\ \hline 942 \end{array}$$

Next, multiply 314 by the number in the tens place of 203. Since 0 is in the tens place of 203 and 0 times any number is just 0 we get

$$\begin{array}{r} 314 \\ \times\ 203 \\ \hline 942 \\ 0000 \end{array}$$

Now multiply 314 by the number in the hundreds place of 203. That is,

$$2(314) = 2(300+10+4) = 600+20+8 = 628.$$

Vertically we get

$$\begin{array}{r} 3\ 1\ 4 \\ \times\ 2\ 0\ 3 \\ \hline 9\ 4\ 2 \\ 0\ 0\ 0\ 0 \\ 6\ 2\ 8\ 0\ 0 \end{array}$$

The last step is to add the results. That is,

$$\begin{array}{r} 3\ 1\ 4 \\ \times\ 2\ 0\ 3 \\ \hline 9\ 4\ 2 \\ 0\ 0\ 0\ 0 \\ +\ 6\ 2\ 8\ 0\ 0 \\ \hline 6\ 3,\ 7\ 4\ 2 \end{array}$$

1.2.6 Division of Whole Numbers

The division of whole numbers can be described as repeated subtraction. Meaning, the statement "15 divided by 3" can be interpreted as asking "how many times does 3 go into 15?" To answer, note the following.

1) $15-3=12$

2) $12-3=9$

3) $9-3=6$

4) $6-3=3$

5) $3-3=0$

Because we had to subtract 3 from 15 five times to get to zero, the result of dividing 15 by 3 is 5.

We can also use what we know about multiplication to answer division questions. We just showed that 15 divided by 3 is 5. However, since we also know that $15 = 5 \cdot 3$, it must be that 3 goes into 15 five times and thus, 15 divided by 3 is 5.

The operation of division is denoted with the symbol $\div$. Moreover, in division, the number being divided is called the **dividend** and the number doing the dividing is called the **divisor**. The result of the division is called the **quotient**.

As an example, in the statement $108 \div 12 = 9$ (108 divided by 12 is 9), the dividend is 108, the divisor is 12, and the quotient is 9.

Just as with the other operations we have discussed we can use different terminology to describe the operation of division. In other words, the symbolic statement $28 \div 4$ can be written in English as

- 28 divided by 4.
- Divide 28 by 4.
- The quotient of 28 and 4.
- Divide 4 into 28.

The next couple of examples illustrate the idea of using multiplication to simplify expressions involving division.

However, before you study the examples, there is an important aspect of division that you need to remember.

Helpful Hint: Division by Zero is Undefined

Recall that any number a times 0 is 0. So for any non-zero numbers a and b,

$$a \div 0 = b \quad \text{would imply that} \quad a = b \cdot 0.$$

This would mean that $a = 0$ which is not a true statement so division by zero is undefined.

The next two examples illustrate the use of multiplication to simplify expressions involving division.

EXAMPLE 1 Simplify the expression $\frac{24}{6}$.

SOLUTION: To simplify this expression, note that because

$$6 \cdot 4 = 24,$$

we know that "*four six's*" go into the number 24. Thus,

$$\frac{24}{6} = 4.$$

Note that while you may be tempted to grab your calculator every time you need to simplify expressions, it will help you to be successful in this and subsequent courses if you remember how multiplication and division work.

EXAMPLE 2 Simplify the expression $\frac{56}{7}$.

SOLUTION: To simplify this expression, note that because $7 \cdot 8 = 56$, we know that "*eight seven's*" go into the number 56. Thus,

$$\frac{56}{7} = 8.$$

EXAMPLE 3 A tank holds 200 gallons of water. How many 50-gallon barrels can be filled from the tank?

SOLUTION: To answer this question we need to know how many times 50 goes into 200. We can accomplish this by dividing 200 by 50. That is, the number of 50-gallon barrels that can be filled is given by

$$200 \text{ gallons} \div 50 \text{ gallons per barrel} = 4 \text{ barrels}.$$

A 200-gallon tank can fill four 50-gallon barrels.

Helpful Hint: Dividing More Than Two Numbers

In order to calculate the quotient of three or more numbers we can begin on the left by dividing two numbers at a time. For example,

$$120 \div 3 \div 2 \div 5 = (120 \div 3) \div 2 \div 5 = 40 \div 2 \div 5 = (40 \div 2) \div 5 = 20 \div 5 = 4.$$

A Number Divided by Itself

For a non-zero number a, $a \cdot 1 = a$ implying any number divided by itself must be 1.

$$\frac{a}{a} = 1,\ a \neq 0.$$

EXAMPLE 4 Simplify the expression $30 \div 2 \div 3 \div 5$.

SOLUTION: Working from left to right we get

$$30 \div 2 \div 3 \div 5$$

$$= (30 \div 2) \div 3 \div 5$$

$$= 15 \div 3 \div 5$$

$$= (15 \div 3) \div 5$$

$$= 5 \div 5$$

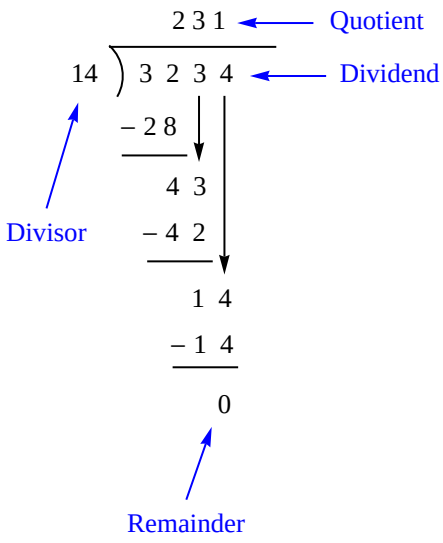
$$= 1$$

EXAMPLE 5 Simplify the expression $42 \div 7 \div 2$.

SOLUTION: Working from left to right we get

$$42 \div 7 \div 2 = (42 \div 7) \div 2 = 6 \div 2 = 3$$

Using knowledge of multiplication to simplify the division of two numbers works well when the two numbers are relatively small. When the numbers involved in the division start to become larger we may need to rely on what is called **Long Division**. To illustrate the use of long division, divide 3234 by 14.



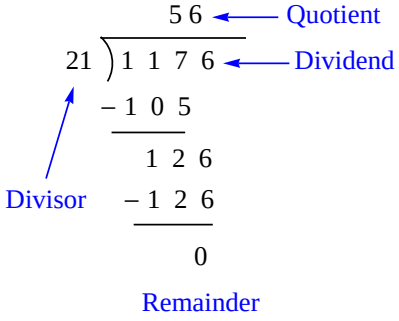
The basic steps for long division are:

1. 14 goes into 32 evenly 2 times
2. Multiply $2 \cdot 14 = 28$
3. Subtract $32 - 28 = 4$
4. Bring down the next number, 3, and then repeat steps 1 - 4 with the new dividend.

In general, this process is repeated until the dividend is smaller than the divisor. Since we are working with Whole Numbers, this means that we repeat until we obtain a remainder of zero.

EXAMPLE 6 Divide 1176 by 21.

SOLUTION:



1. 21 goes into 117 evenly 5 times
2. Multiply $5 \cdot 21 = 105$
3. Subtract $117 - 105 = 12$
4. Bring down the next number, 6.
5. 21 goes into 126 evenly 6 times
6. Multiply $6 \cdot 21 = 126$
7. Subtract $126 - 126 = 0$

The remainder is zero, so the result is $\frac{1176}{21} = 56$.

1.2.7 Order of Operations: Multiplication and Division of Whole Numbers

Combining the Helpful Hints from throughout the section gives us rules on how to simplify expressions involving whole number exponents, and multiplication and division.

Order of Operations
Multiplication and Division of Whole Numbers
Simplify any expressions with exponents.
Perform Multiplication and Division as they occur from left to right.

EXAMPLE 1 Simplify the expression $4 \cdot 6 \div 3 \cdot 2^3 \div 16$

SOLUTION: Order of Operations requires that we simplify any expressions involving exponents first. Because this expression contains 2^3, we must take care of this first.

$$2^3 = 2(2)(2) = 8.$$

The original expression can now be written as

$$4 \cdot 6 \div 3 \cdot 2^3 \div 16 = 4 \cdot 6 \div 3 \cdot 8 \div 16.$$

Performing multiplication and division as they occur from left to right gives the following.

$$4 \cdot 6 \div 3 \cdot 8 \div 16 = (4 \cdot 6) \div 3 \cdot 8 \div 16$$

$$24 \div 3 \cdot 8 \div 16$$

$$= (24 \div 3) \cdot 8 \div 16$$

$$= 8 \cdot 8 \div 16$$

$$= (8 \cdot 8) \div 16$$

$$= 64 \div 16$$

$$= 4$$

EXAMPLE 2 Simplify the expression $24 \div 4 \cdot 3^2 \cdot 4 \div 2^3$.

SOLUTION: Using Order of Operations we get

$$24 \div 4 \cdot 3^2 \cdot 4 \div 2^3 = 24 \div 4 \cdot 9 \cdot 4 \div 8$$

$$= (24 \div 4) \cdot 9 \cdot 4 \div 8$$

$$= (6 \cdot 9) \cdot 4 \div 8$$

$$= (54 \cdot 4) \div 8$$

$$= 216 \div 8$$

$$= 27$$

Section 1.2 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 3, round the given number to the indicated place.

1) Round 167 to the tens place.

2) Round $45,639$ to the hundreds place.

3) Round $6,588,275$ to the thousands place.

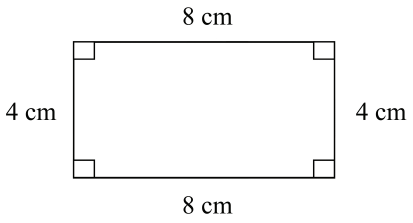
For problems 4 - 5, identify the factors and the product.

4) $6 \times 8 = 48$

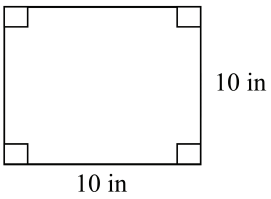
5) $3 \cdot 5 \cdot 8 = 120$

For problems 6 - 7, find the area of the given figure.

6)



7)



For problems 8 - 10, evaluate the given expression.

8) 4^3

9) 5^2

10) 10^5

For problems 11 - 13, use the distributive property to perform the indicated multiplication.

11) $5(22)$

12) $(15)(3)$

13) $4(93)$

For problems 14 - 16, perform the multiplication by multiplying the numbers vertically.

14) $55(49)$

15) $121(38)$

16) $58(173)$

For problems 17 - 18, identify the dividend, divisor, and quotient.

17) $108 \div 12 = 9$

18) $\dfrac{55}{11} = 5$

For problems 19 - 21, perform the indicated division.

19) $36 \div 9$

20) $\dfrac{24}{6}$

21) $45 \div 5$

For problems 22 - 24, perform the division using Long Division.

22) $1625 \div 13$ 23) $4842 \div 9$ 24) $12,663 \div 21$

For problems 25 - 29, use Order of Operations to simplify the expression.

25) $4 \cdot 5 \cdot 3 \div 6$ 26) $48 \div 8 \div 6 \cdot 11$ 27) $4^2 \cdot 3 \div 12$

28) $15 \div 3 \cdot 4 \div 2 \cdot 3^3$ 29) $28 \div 7 \cdot 5^2 \div 25$

For problems 30 - 32, solve the given applications involving multiplication.

30) One ream of paper has 500 pages. A case of paper contains 5 reams. How many pages are in a case of paper?

31) Franks SUV gets 18 miles per gallon. If the tank of the SUV holds twenty gallons, how far can Frank travel on a full tank of gas?

32) A rain barrel holds 50 gallons of water. How many gallons are stored in 15 rain barrels?

For problems 33 - 35, solve the given applications involving division.

33) How many six packs can be made from 288 cans?

34) If 18 cups of flour can make 9 cakes, how many cups are needed to make one cake?

35) Two hundred and ten people are attending an event. How many 6 packs of cola will be needed in order to ensure that each person receives a drink?

Chapter 1
Section 1.2 – Rounding, Multiplication and Division of Whole Numbers and Calculating Area

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

1) _____

2) _____

3) _____

4) Factors: _____Product: _____

5) Factors: _____Product: _____

6) _____

7) _____

8) _____

9) _____

10) _____

11) _____

12) _____

13) _____

14) _____

15) _____

16) _____

17) Dividend: _____Divisor: _____

Quotient: _____

18) Dividend: _____Divisor: _____

Quotient: _____

18) _____

19) _____

20) _____

21) _____

22) _____

23) _____

24) _____

25) _____

26) _____

27) _____

28) _____

29) _____

30) _____

31) _____

32) _____

33) _____

34) _____

35) _____

1.3 Order of Operations

1.3.1 Order of Operations: Addition, Subtraction, Multiplication, and Division

Combining the Order of Operations for Addition and Subtraction, along with Exponents, Multiplication and Division into a more general Order of Operations gives the following.

Quick Note

Parentheses and brackets are commonly used grouping symbols. In future sections the numerators and denominators of fractions as well as absolute values and radicals will also be grouping symbols.

Order of Operations

1) Perform all operations inside parentheses, or any grouping symbols, first.

2) Simplify any expressions containing exponents.

3) Perform multiplication and division as they occur from left to right.

4) Perform addition and subtraction as they occur from left to right.

EXAMPLE 1 Simplify the expression $45 \div 9 + 3 - 2(3) + 4$.

SOLUTION: Notice that even though there is a 3 within parentheses, it cannot be simplified any more than it is. Moreover, since there aren't any exponents to evaluate we move on to steps 3 and 4 of the Order of Operations.

Multiplication and Division as they occur from left to right.

$$45 \div 9 + 3 - 2(3) + 4$$

$$= 5 + 3 - 6 + 4$$

Addition and Subtraction as they occur from left to right.

$$= 5 + 3 - 6 + 4$$

$$= 8 - 6 + 4$$

$$= 8 - 6 + 4$$

$$= 2 + 4$$

$$= 2 + 4$$

$$= 6.$$

Therefore, $45 \div 9 + 3 - 2(3) + 4 = 6$.

EXAMPLE 2 Simplify the expression $5 + 8(6 - 2^2) + 18 \div 3^2 - 1$.

SOLUTION: According to the Order of Operations, the first step is to perform any operations within any grouping symbols.

Groupings

When performing operations within grouping symbols we follow the Order of Operations. So here,

$$5 + 8(6 - 2^2) + 18 \div 3^2 - 1$$

we need to begin by simplifying $6 - 2^2$.

Within the grouping symbols the first step is to take care of any exponents. This means we have

$$5 + 8(6 - 2^2) + 18 \div 3^2 - 1 = 5 + 8(6 - 4) + 18 \div 3^2 - 1.$$

There is no multiplication or division indicated inside of the parentheses so we move on to calculating addition and subtraction as they occur from left to right. Therefore,

$$5 + 8(6 - 4) + 18 \div 3^2 - 1 = 5 + 8(2) + 18 \div 3^2 - 1.$$

Now that the expressions within all of the groupings have been accounted for, we step our way through the rest of the expression using the Order of Operations .

Exponents

$$5+8(2)+18\div 3^2-1=5+8(2)+18\div 9-1.$$

Multiplication and Division as they occur from left to right.

$$5+8(2)+18\div 9-1=5+16+2-1.$$

Addition and Subtraction as they occur from left to right.

$$5+16+2-1=21+2-1=23-1=22.$$

EXAMPLE 3 Simplify the expression $2[18-2(4\{3-1\})]\div 2$.

SOLUTION: According to the Order of Operations we need to perform all operations inside any grouping symbols first. Because this problem has multiple grouping symbols we begin within the **inner** most grouping and work our way out.

$$2[18-2(4\{3-1\})]\div 2=2[18-2(4\{3-1\})]\div 2$$

$$=2[18-2(4\{2\})]\div 2$$

$$=2[18-2(4\{2\})]\div 2$$

$$=2[18-2(8)]\div 2$$

$$=2[18-2(8)]\div 2$$

$$=2[18-16]\div 2$$

$$= 2[18 - 16] \div 2$$

$$= 2[2] \div 2$$

$$= 4 \div 2$$

$$= 2$$

So, $2[18 - 2(4\{3 - 1\})] \div 2 = 2$.

EXAMPLE 4 Simplify the expression $6 + 2[4(5-2)^2 \div 12]^3 - 28$.

SOLUTION: Starting within the innermost grouping and working our way out using the Order of Operations we get the following.

$$6 + 2[4(5-2)^2 \div 12]^3 - 28 = 6 + 2[4(5-2)^2 \div 12]^3 - 28$$

$$= 6 + 2[4(3)^2 \div 12]^3 - 28$$

$$= 6 + 2[4(9) \div 12]^3 - 28$$

$$= 6 + 2[36 \div 12]^3 - 28$$

$$= 6 + 2[3]^3 - 28$$

$$= 6 + 2[27] - 28$$

$$= 6 + 54 - 28$$

$$= 60 - 28$$

$$= 32.$$

EXAMPLE 5 Simplify the expression $3\left[4+(9-5)^2 \div 8\right]+6(3^3-5^2) \div 4$.

SOLUTION: Starting within the innermost grouping and working our way out using the Order of Operations we get the following.

$$3\left[4+(9-5)^2 \div 8\right]+6(3^3-5^2) \div 4$$

$$= 3\left[4+(4)^2 \div 8\right]+6(27-25) \div 4$$

$$= 3\left[4+16 \div 8\right]+6(2) \div 4$$

$$= 3\left[4+16 \div 8\right]+6(2) \div 4$$

$$= 3\left[4+2\right]+12 \div 4$$

$$= 3\left[4+2\right]+12 \div 4$$

$$= 3\left[6\right]+3$$

$$= 3\left[6\right]+3$$

$$= 18+3$$

$$= 21.$$

Section 1.3 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For 1 - 12, use the Order of Operations to simplify the expression.

1) $16 \div 4(3) \div 6$

2) $12 - 10 \div 5 + 8 \div 4$

3) $(15+6) \div 7(2+4)$

4) $4^2 - 8 + 2(13-9)$

5) $15 \div 3 \div 5 + 2(6-1)^2$

6) $55 \div 11 + 3(4) - 6$

7) $8 + 2[18 - 3(2+1)] \div 3 - 10$

8) $(16 \div 8(5) - 6)^3 - 5^2$

9) $5 + [6 + (4 + (7+1)) + 2]$

10) $81 \div 3 \div 9 \div 3$

11) $[40 - 8(7-5)^2 - 5]^4 - 10$

12) $100 \div 10 - 9[46 - 5(11-8)^2]^3$

For 13 - 18, perform the multiplication by multiplying the numbers vertically.

13) $53(29)$

14) $37(7)$

15) $91(18)$

16) $12(351)$

17) $504(6)$

18) $27(635)$

For 19 - 24, perform the division using Long Division.

19) $793 \div 13$

20) $594 \div 18$

21) $1089 \div 9$

22) $432 \div 8$

23) $5133 \div 87$

24) $3348 \div 54$

25) John's salary was \$800 dollars a week. After a stellar progress report, he received a raise and his salary increased by \$200. What is his new salary after the raise?

26) Rebecca earned 97, 88, 56, and 76 points on 4 quizzes. What is the total amount of points earned on the four quizzes?

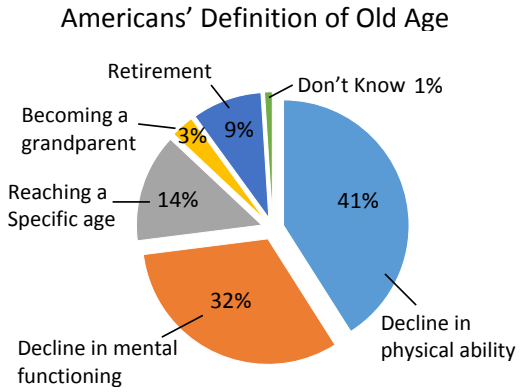
27) The table gives the number and make of automobiles sold at an auction for the month of January. Find the number of total cars sold in January.

Make	Honda	Ford	Chevrolet	Nissan	Chrysler
Number Sold	150	25	75	90	49

28) A grocery bill was \$76 and \$100 was given to the cashier. Find the amount of change received.

29) A class decided to have a pizza party. The pizzas cost $36. Before the pizza arrived they collected $25. How much more money do they need to pay the bill in full.

For problems 30 **and** 31 **use the figure below. The pie graph shows the percentage of what people think old age is.**



30) What percentage of people think that old age is other than becoming a grandparent?

31) What percentage of people think that old age is becoming a grandparent or decline in mental functioning?

Mixed Practice. First translate the English phrase into an equivalent mathematical statement then simplify the statement.

32) Five more than 23.

33) Three less than 14.

34) The sum of 18 and 25.

35) The difference of 103 and 91.

Chapter 1
Section 1.3 – Order of Operations

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

1) _____

2) _____

3) _____

4) _____

5) _____

6) _____

7) _____

8) _____

9) _____

10) _____

11) _____

12) _____

13) _____

14) _____

15) _____

16) _____

17) _____

18) _____

19) _____

20) _____

21) _____

22) _____

23) _____

24) _____

25) _____

26) _____

27) _____

28) _____

29) _____

30) _____

31) _____

32) _____

33) _____

34) _____

35) _____

1.4 Problem Solving

1.4.1 One-Step Applications

Knowing how operations are expressed in English will help us to understand how to set up and solve applications. The table provided below gives some common terminology that will be used for basic arithmetic operations.

Common Terminology for Basic Arithmetic Operations	
Addition: $6+2$	Subtraction: $6-2$
6 plus 2 The total of 6 and 2 The sum of 6 and 2 2 added to 6 2 more than 6	6 minus 2 6 subtract 2 6 less 2 The difference of 6 and 2 2 subtracted from 6 2 less than 6
Multiplication: $6 \cdot 2$ or 6×2	Division: $6 \div 2$ or $\frac{6}{2}$
6 times 2 The product of 6 and 2 6 multiplied by 2	6 divided by 2 The quotient of 6 and 2 2 divided into 6

EXAMPLE 1 Aja needs $850 to pay her tuition. She has saved $500. How much more will she need to pay her tuition?

SOLUTION: The use of the phrase "how much more" implies that the amount she has is not enough. This means that we should find the difference between how much she needs and how much she has. That is,

$$\text{Amount she needs} - \text{Amount she has} = \text{Amount Needed}.$$

Replacing the words with the amounts given in the problem gives

$$\$850 - \$500 = \text{Amount Needed}.$$

Simplifying this expression gives $350. Therefore, the answer is Aja needs an additional $350 to pay her tuition.

Phrases such as "how much more" and "how many more" imply subtraction may be needed to solve the problem. Also, phrases such as "how much higher" or "how much lower" can indicate that the difference between given quantities must be found. In these circumstances we typically subtract the smaller value from the larger value.

In the next few examples we take a look at applications that involve subtraction and the word "per".

EXAMPLE 2 Tia makes $10 per hour at her part time job. Tamara makes $14 per hour at her part time job. What is the difference in their hourly earnings?

SOLUTION: Because the problem is asking for the "difference" in their hourly earnings we know we must use subtraction to solve the problem. That is,

$$\text{Higher hourly wage} - \text{Lower hourly wage} = \text{Difference in earnings},$$

$$\Rightarrow \text{Tamara's earnings} - \text{Tia's earnings} = \text{Difference in earnings}.$$

Replacing the words with the amounts given in the problem and simplifying gives

$$\$14 - \$10 = \$4.$$

This means that the difference in their earnings is $4 per hour.

EXAMPLE 3 Sameer earns $9 per hour. If he works 32 hours in one week, what is his pay before taxes?

SOLUTION: The word "per" is used to indicate one unit over some another unit. An everyday example would be

$$\text{miles "per" gallon, meaning } \frac{\text{miles}}{\text{gallon}}$$

or in the case of this example and the last,

$$\text{dollars "per" hour, meaning } \frac{\text{dollars}}{\text{hour}}.$$

In this problem we are given dollars per hour and asked to find total pay after 32 hours. To calculate this we multiply the amount of dollars per hour by the number of hours.

$$\$9 \text{ dollars per hour} \times 32 \text{ hours} = \$288.$$

Sameer earns $288 before taxes working 32 hours.

Note: Had the total earnings been given instead of the number of hours worked we would have needed to divide the total earnings by the per hour rate to get the number of hours worked.

EXAMPLE 4 Sameer earns $9 per hour. In one week he earned $261 before taxes. How many hours did he work during that week?

SOLUTION: As mentioned in the previous example, we need to divide the total earned in one week by the hourly rate. That is

$$\text{Total earned} \div \text{Hourly rate} = \text{Number of hours worked.}$$

Replacing the words with the numbers given in the problem and simplifying we get

$$\$261 \div \$9 \text{ per hour} = 29 \text{ hours.}$$

Sameer worked 29 hours that week.

1.4.2 Multi-step Applications

Solving some applications may require that we solve another application before we can solve for the requested answer. The next few examples deal with solving applications that involve more than one step.

EXAMPLE 1 Margaret's car has a 14-gallon tank. She gets 40 miles per gallon driving on the highway. How many times will she need to fill her tank to drive 2,240 miles?

SOLUTION: We start the solution to this problem by calculating the number of miles she can drive on one tank of gas. This means we need to calculate

$$\text{miles per gallon} \times \text{number of gallons per tank} = \text{Total miles per tank.}$$

Replacing the words with the values given in the problem and simplifying gives

$$40 \text{ miles per gallon} \times 14 \text{ gallons per tank} = 560 \text{ miles per tank.}$$

Avoiding Mistakes

It is always a good idea to check your answer in the context of the problem to determine if your answer is reasonable.

Now, divide the total number of miles by the number of miles per tank.

$$2{,}240 \text{ miles} \div 560 \text{ miles per tank} = 4 \text{ tanks.}$$

Margaret will need to fill her tank 4 times to drive 2, 240 miles.

ALTERNATIVELY: For this problem, we could first find the number of gallons she needs to drive 2,240 miles, and then calculate the number of tanks that would be needed.

Calculate the number of gallons she needs for the trip.

$$\text{Total miles} \div \text{miles per gallon} = \text{Total number of gallons}$$

$$\Rightarrow \text{ 2,240 miles} \div 40 \text{ miles per gallon} = 56 \text{ gallons.}$$

Now divide the total gallons of gas needed by the number of gallons the tank will hold.

$$56 \text{ gallons} \div 14 \text{ gallons per tank} = 4 \text{ tanks.}$$

Margaret will need to fill her tank 4 times to drive 2,240 miles.

EXAMPLE 2 Kadisha's grocery bill is \$113. How many \$20 bills will she need to pay her bill? How much change will she get back if she pays her bill with \$20 bills?

SOLUTION: To determine how many \$20 bills she will need note that 20 goes into 113 five times with a remainder of 13. Because there is a remainder, she will need six \$20 dollar bills to pay the grocery bill.

Since she needs six \$20 bills, she will end up paying $6 \times \$20 = \$120$. To find the amount of change she will get back, subtract \$113 from \$120.

$$\$120 - \$113 = \$7$$

Kadisha will need six \$20 dollar bills to pay the bill of \$113 and she will receive \$7 in change.

1.4.3 Applications Involving Geometric Formulas

Some applications involve the use of geometric formulas. To help solve these problems it is recommended that we pay attention to the units being used.

Applications involving perimeter use linear units, and applications involving area use square units (see section 1.2.2. example 3). Because of this, the units may give a clue towards choosing the right type of formula.

Common Geometric Formulas	
Formulas for Perimeter	**Formulas for Area**
Triangle with sides a, b, and c: $P = a+b+c$	**Triangle** with base b and height h: $A = \frac{1}{2}bh$
Square with sides s: $P = 4s$	**Square** with sides: $A = s \cdot s$ (or) $A = s^2$
Rectangle with length l and width w: $P = 2l+2w$	**Rectangle** with length l and width w: $A = l \cdot w$

EXAMPLE 1 Alex has 120 m of fencing to enclose a rectangular garden in his yard. His yard is 35 m long. What is the widest rectangle he can enclose with the 120 m of fencing?

SOLUTION: The first step is to determine whether we need to use a formula for perimeter or a formula for area. Since we are given the total length of the fence (the fence is 120 m), or perimeter, we should use a perimeter formula. The perimeter of a rectangle with length l and width w is $P = 2l+2w$. We are given the perimeter and the length and asked to find the width. This implies that we should use subtraction since $P-2l = 2w$.

$$P-2l = 2w$$

Putting in the values given in the statement of the problem we get

$$120\,\text{m} - 2(35\,\text{m}) = 2w \;\Rightarrow\; 120\,\text{m} - 70\,\text{m} = 2w$$

Avoiding Mistakes

Units of measurement must be included for the answer to be complete.

$$\Rightarrow \quad 50\,\text{m} = 2w.$$

Dividing the 50 m by 2 gives a width of 25 m. So the widest rectangle that Alex can make is 25 m.

EXAMPLE 2 Sara's patio floor is a rectangle that is 15 ft long by 6 ft wide. How many square tiles with sides of 3 ft can she place on the her floor?

SOLUTION: The first step is to determine the area of the floor. Since the floor is a rectangle we use the formula for the area of a rectangle. That is,

$$\text{Area of the floor} = (\text{length of the floor}) \cdot (\text{width of the floor}).$$

Putting in the values given in the statement of the problem we get

$$\text{Area of the floor} = (15\text{ ft}) \cdot (6\text{ ft}) = 90\text{ ft}^2.$$

Next, find the area of one of the tiles. Since the tiles are square with sides of length 3 ft,

$$\text{Area of one tile} = (3\text{ ft}) \cdot (3\text{ ft}) = 9\text{ ft}^2.$$

To determine how many tiles Sara will need, divide the total area of the floor by the area of one tile to get

$$(90\text{ ft}^2) \div (9\text{ ft}^2) = 10.$$

This means that Sara can place 10 square tiles with sides of 3 ft on her patio floor.

Section 1.4 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 6, list the operation for each key word or phrase.

1) More Than

2) Less Than

3) The Product of

4) The Difference of

5) The Quotient of

6) The Sum of

For problems 7 - 25, solve the application.

7) The highest mountain in the world is Mount Everest at 29,029 feet. The second highest mountain is Aconcagua at 22,837 feet. How much higher is Mount Everest than Aconcagua?

8) A garden is used for planting broccoli. If there are 10 rows consisting of 12 broccoli plants, how many total plants are in the garden?

9) The in-state cost of tuition at Daytona State College is approximately $102 per credit hour. The out-of-state cost is approximately $399 per credit hour.

a) How much more will a person attending from out-of-state have to pay per credit hour?

b) If a person is paying in-state tuition, how much would it cost to take 15 credit hours?

10) If a degree takes 15 hours per semester for 4 semesters, how many hours are needed for the degree.

11) 900 points are needed for an **A** and Frank already has 260 points. On average, how many points will he need on each of the next 8 tests to earn an **A**?

12) Frank's SUV gets 18 miles per gallon. The SUV has a 22 gallon tank. How many times will he have fill his tank if he wants to drive 1,980 miles?

13) There are 360 gallons stored in rain barrels. How many rain barrels are there if each one can hold 45 gallons?

14) John saves $125 a week. How many weeks will he need to save in order to make a $2000 down payment.

15) Melissa purchased a house for $\$360,000$. She put $\$72,000$ down and will make even payments for the next 360 month period. How much will be her monthly payments?

16) Henry's kitchen is 12 ft long and 14 ft wide. How many square tiles that have a side of 2 feet will be needed to cover the floor?

17) John has bought a pallet of sod. Each piece of sod is two feet by three feet. How much of his yard can he cover if a pallet holds 100 pieces?

18) Svetlana's rectangular garden is 100 ft by 50 ft. If she wants to put chicken wire around it, how many feet of chicken wire does she need to buy.

19) Referring to number 19, what is the area Svetlana needs to cover the ground with soil.

20) If a room has a length of 20 feet and an area of 300 ft^2, what is its width?

21) If the perimeter of a rectangle surface is 170 ft and the length is 50 ft, what is its width?

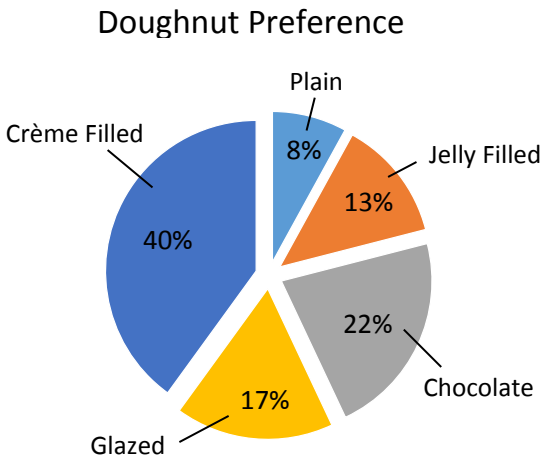
22) Sasha makes $550 in one week before taxes. If she makes $11 per hour, How many hours did she work?

23) A normal sized class at Daytona State College has 4 rows with 10 seats per row. What is the maximum seating capacity of the classroom?

24) Jonathan pays for his $135 grocery bill with $50 bills. How many bills did he use?

Review.

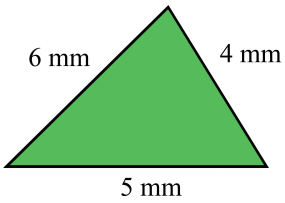
For problems 25 - 26, use the figure below. The pie graph shows the percentage of peoples' doughnut preferences.



25) How many more percent of people prefer chocolate covered doughnuts to glazed doughnuts?

26) How many percent of people prefer glazed doughnuts, chocolate covered doughnuts, or jelly filled doughnuts?

27) Find the perimeter of the triangle



28) What number is 10 more than 11?

29) The product of 2 and 12 will result in what number?

30) The quotient of 99 and 3 results in what number?

31) Is it possible to divide by 0?

32) How many $100 bills can you make out of fifteen $20 bills?

33) If the area of a square is $100\,\text{ft}^2$, what is its perimeter?

34) If Gabe's Honda gets 18 mpg in the city, how far can he travel on a 22 gallon tank?

Chapter 1
Section 1.4 – Problem Solving

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

1) _____

2) _____

3) _____

4) _____

5) _____

6) _____

7) _____

8) _____

9) a) _____

b) _____

10) _____

11) _____

12) _____

13) _____

14) _____

15) _____

16) _____

17) _____

18) _____

19) _____

20) _____

21) _____

22) _____

23) _____

24) _____

25) _____

26) _____

27) _____

28) _____

29) _____

30) _____

31) _____

32) _____

33) _____

34) _____

INTEGERS AND THEIR APPLICATIONS

CHAPTER

2

CHAPTER OBJECTIVES

2.1 Integers and the Real Number Line

2.2 Addition and Subtraction of Intergers

2.3 Multiplication and Division of Integers

The Infinity Symbol

Often the simplest symbols are the ones with the richest meanings.The infinity sign, the figure of eight, and the mathematical lemniscate all refer to the same shape that contains a wealth of complex meaning within its fluid lines. The (infinity sign) is a highly popular math symbol. This could be attributed to the fact that “infinity” symbolizes that which has no ending, one that lasts forever. As a mathematical device, the infinity sign was first "discovered" in 1655 by John Wallis, but its significance as a religious symbol is much older. The infinity sign has its origins in the Arabic numerals that actually came from India in the first place. The sign can be drawn in one continuous movement, making a seesaw movement of clockwise and counterclockwise loops. These loops reflect the balance of opposites: male and female, day and night, dark and light. The symbol was a deviation of an ancient symbol depicting a dragon chasing and wanting to eat its very own tail. People tend to be drawn to the idea of infinity as it could relate to infinite love, wealth and all things good.

2.1 Integers and the Real Number Line

In Chapter 1 we worked with the set of Whole Numbers. The set of Whole Numbers consisted of the numbers

$$0, 1, 2, 3, \ldots.$$

This set contains the number 0 and all of the "counting numbers" larger, or greater, than zero. We refer to the numbers greater than zero as **positive numbers**.

In this chapter we begin working with the set of **Integers**. The set of Integers contains the set of Whole Numbers as well as the "**negatives**" of the whole numbers. That is, the set of Integers contains the numbers

$$\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots .$$

Note that since positive numbers are numbers that are larger, or greater than zero, **negative numbers** are numbers that are smaller, or less than zero.

Just as in the first chapter we will investigate the operations of addition, subtraction, multiplication and division of integers. We begin our investigation by giving some examples of how negative whole numbers can be used.

The use of negative numbers in football.

Consider a football team that starts on its own 20 yard line. On first down, they run the ball and the running back makes it out to the 23 yard line before he is tackled. The yardage on the first down is then $+3$ yards, or just 3 yards. On second down, the team attempts a pass and the pass is incomplete. The ball goes back to where they started on the 23 yard line. The yardage on the second down is then 0 yards. On third down, the team attempts to pass the ball. The quarterback is tackled 5 yards behind where the ball started. The yardage on third down would then be -5 yards. That is, they went back 5 yards instead of forward 5 yards, and thus the yardage is -5 (negative 5) yards.

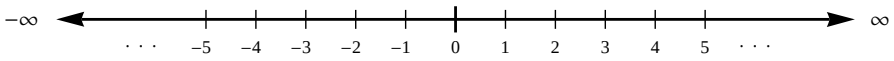
The use of negative numbers in expressing altitude.

Another example of how negative numbers are used is in describing the altitude of various locations on the planet. We refer to "sea level" as zero on the scale of altitude. Mount Rushmore's altitude is $5,725$ feet above sea level while Death Valley is 282 feet below sea level. Because Death Valley's altitude is measured below sea level, we say its altitude is -282 feet.

2.1.1 The Real Number Line

The **Real Number Line** is a line on which every point represents a Real Number. We emphasize integer values on the number line, but it is important to remember every single point on the Real Number Line represents a Real Number. That is, fractions and decimals that fall between integer values are also on the Real Number Line.

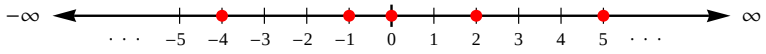
Positive Real Numbers are located to the right of zero, and negative numbers are located to the left of zero on the Real Number Line. A typical representation of the Real Number Line is shown here.



Note the use of the dots .... These indicate that the positive Real Numbers keep increasing to infinity (∞) to the right and the negative numbers keep decreasing to negative infinity ($-\infty$) to the left.

EXAMPLE 1 Graph the integers $\{-4,-1,0,2,5\}$ on the Real Number Line.

SOLUTION:



2.1.2 Inequalities

One rather important property of the Real Number Line is that it orders the Real Numbers.

All numbers that fall to the right of some given number are considered to be larger than the given number. For example, we know that 12 and 56 are larger numbers than 7. This is indicated on the Real Number Line by the fact that 12 and 56 occur to the right of 7.

Likewise, all numbers that fall to the left of some given number are considered to be smaller than the given number. For instance, -6 and -23 are considered to be smaller than -3 and are therefore graphed to the left of -3 on the Real Number Line.

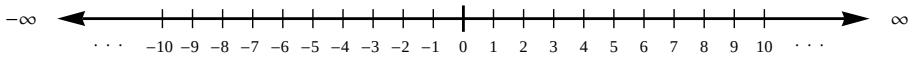
This "ordering" of numbers is denoted, or expressed, with **inequalities**. In this section we consider the inequalities $<$ and $>$.

The statement "$m > n$", read as "m is greater than n", is true if and only if m lies to the right of n on the Real Number Line. In other words, $m > n$ is true only if m is actually a larger value that n.

The statement "$p < q$", read as "p is less than q", is true if and only if p lies to the left of q on the Real Number Line.

The next three examples illustrate how the inequalities $<$ and $>$ are used to express which of two numbers is larger, or smaller, than the other.

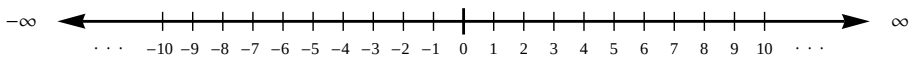
EXAMPLE 1 Insert the inequality $<$ or $>$ to make the statement "8 $\square$ 5" true. If need be use the Real Number Line for reference.



SOLUTION: On the Real Number Line we see that 8 lies to the right of 5. Therefore,

$$8 > 5.$$

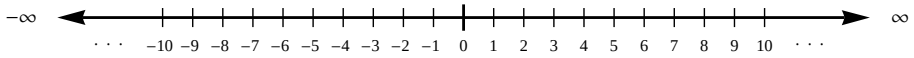
EXAMPLE 2 Insert the inequality $<$ or $>$ to make the statement "-4 $\square$ 9" true. If need be use the Real Number Line for reference.



SOLUTION: On the Real Number Line, every negative number lies to the left of every positive number, so it is safe to say that every negative number will be less than every positive number. Therefore,

$$-4 < 9.$$

EXAMPLE 3 Insert the inequality $<$ or $>$ to make the statement "$-2 \square -7$" true. If need be use the Real Number Line for reference.



SOLUTION: Referring to the Real Number Line again, we see that -2 lies to the right of -7. Therefore,

$$-2 > -7.$$

2.1.3 Negative Signs: "The Opposite of ...", and Absolute Value

Negative Signs: "The Opposite of ...":

The symbol "$-$" can have different interpretations depending on the context in which it is used. We said in chapter 1 that it can represent subtraction. That is, $7-2$ indicates that we should subtract 2 from 7.

We can also use the symbol $-$ to indicate "the opposite of" a number.

The opposite of any given number is found by simply changing its sign from negative to positive, or from positive to negative. For example,

$$\text{“the opposite of” } 6 = -6, \text{ and “the opposite of” } -3 = -(-3) = 3.$$

We typically refer to the opposite of a number as its “negative.” With the example this means

$$\text{“the opposite of” } 6 = \text{“negative” } 6 = -6$$

and

$$\text{“the opposite of” } -3 = \text{“negative” } -3 = -(-3) = 3.$$

Absolute Value

The Absolute Value of a number is the “magnitude” or “size” of the number. We denote the Absolute Value of the number x as $|x|$ and many times we think of it as the distance from x to zero on the Real Number Line.

As an example, the Absolute Value of 12 is

$$|12| = 12$$

since the number 12 is twelve units away from zero.

Furthermore, the Absolute Value of -8 is

$$|-8| = 8$$

since the number -8 is eight units away from zero.

Note that the position of the number -8 is eight units to the left of zero on the Real Number Line but the Absolute Value of the number -8, its magnitude, is 8.

The concept of the magnitude of a number leads to the actual definition of Absolute Value.

Absolute Value

For any Real Number x,

$$|x| = \begin{cases} x \text{ if } x > 0 \text{ or } x = 0 \\ -x \text{ if } x < 0 \end{cases}$$

EXAMPLE 1 Evaluate the expression $|9|$.

SOLUTION: Since $9 > 0$, by definition, the Absolute Value of 9 is just 9. So,

$$|9| = 9.$$

EXAMPLE 2 Evaluate the expression $|-15|$.

SOLUTION: Since $-15 < 0$, by definition, the Absolute Value of -15 is the opposite of -15. So,

$$|-15| = -(-15) = 15.$$

EXAMPLE 3 Evaluate the expression $-|2|$.

SOLUTION: We can evaluate this one in steps. We start with $|2|$. Since $2 > 0$,

$$|2| = 2.$$

This means that

$$-|2| = -(2) = -2.$$

EXAMPLE 4 Evaluate the expression $-|-23|$.

SOLUTION: We can evaluate this one is steps. We start with $|-23|$. Since $-23 < 0$,

$$|-23| = -(-23) = 23.$$

This means that

$$-|-23| = -(23) = -23.$$

We studied the Order of Operations in the last chapter. If we have an expression involving absolute values, treat the absolute value as a grouping. Consider the following example.

EXAMPLE 5 Simplify the expression $(|(12-9)^3 \div 9 - 5|] + 7)^2$.

SOLUTION: Treating the absolute value as a grouping symbol, using the Order of Operations gives

$$(|(12-9)^3 \div 9 - 5|] + 7)^2 = (|(3)^3 \div 9 - 5|] + 7)^2$$

$$= (|27 \div 9 - 5|] + 7)^2$$

$$= (|3-5| + 7)^2$$

$$= (|-2| + 7)^2$$

$$= (-(-2) + 7)^2$$

$$= (2+7)^2$$

$$= 9^2$$

$$= 81.$$

Section 2.1 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 4, write an integer that represents the given statement.

1) The temperature is 4 degrees below zero.

2) The mountain resort is 5210 feet above sea level.

3) Mary wrote a check for $43.78.

4) Alyssa gained 3 pounds on vacation last week.

For problems 5 - 9, graph the set of integers on the Real Number Line.

5) $\{2,4,6\}$ 6) $\{-6,-4,-2\}$ 7) $\{-4,0,5,6\}$

8) $\{-3,0,3\}$ 9) $\{-6,-2,1,5,9\}$

For problems 10 - 14, insert the inequality $<$ or $>$ to make the statement true.

10) $-3 \square -4$ 11) $3 \square 10$ 12) $-15 \square 0$

13) $12 \square -3$ 14) $-6 \square -5$

For problems 15 - 19, evaluate the Absolute Value Expressions.

15) $|-3|$ 16) $|54|$ 17) $|0|$

18) $|-(-23)|$ 19) $-|-17|$

For problems 20 - 21, state the real number represented by the statement.

20) The opposite of negative 7.

21) The absolute value of 10.

Mixed Practice. For problems 22 - 25, state whether the given statement is True or False.

22) $-|-8| = 8$ 23) $-(-8) = 8$ 24) $|-3| = 3$

25) $-(6) = -6$

Review. For problems 26 - 35, simplify the expression.

26) $10+5$

27) $13-4$

28) $2,359+75,429$

29) $27,540-3,657$

30) $12(24)$

31) $540\div 20$

32) $540(20)$

33) $50(4+81)\div[10(28-3)]$

34) $3+7|2-8|$

35) $|(60+5)-150|\div(10+7)$

Chapter 2
Section 2.1 – Integers and the Real Number Line

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

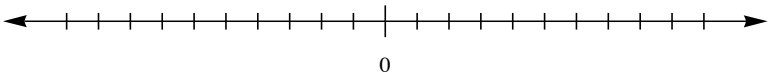
1) _____

2) _____

3) _____

4) _____

5)



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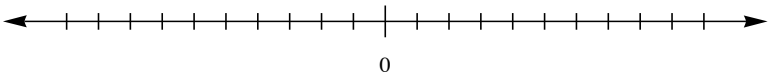

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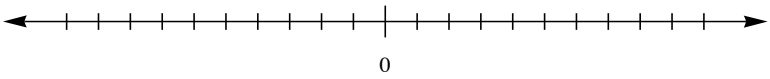

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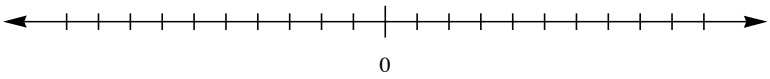

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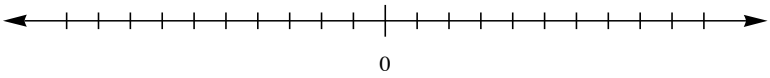

10) _____

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33) _____

34) _____

35) _____

2.2 Addition and Subtraction of Integers

2.2.1 Addition of Integers with the same Sign

The Addition of Integers with the Same Sign
Two Integers with the same sign are added by:
1) adding their absolute values and then
2) writing the sum with the sign common to both numbers.

In Chapter 1 we used a number line to add whole numbers. That is, to perform the addition $5+6$, we could start at 5 on the number line and move to the right 6 units to 11 and so, $5+6=11$. The new thing that we have to deal with in this section is how to handle sums of the form $(-5)+(-6)$. According to the rule above, we find the sum, $(-5)+(-6)$, by adding the absolute values of the -5 and -6 and then assigning their common sign. That is,

$$|-5|+|-6|=5+6=11$$

and since both integers were negative, $(-5)+(-6)=-11$.

Quick Note

The sum of two positive numbers will always be positive and the sum of two negative numbers will be negative.

EXAMPLE 1 Simplify the expression $12+33$.

SOLUTION: Notice that this is exactly the kind of sum we discussed in Chapter 1. That is, since both integers are positive, this is just adding two whole numbers so

$$12+33=35.$$

EXAMPLE 2 Simplify the expression $-9+(-15)$.

SOLUTION: For this problem both numbers are negative so we need to first find the absolute value of each.

$$|-9| = -(-9) = 9, \text{ and } |-15| = -(-15) = 15.$$

Next, we add their absolute values.

$$9 + 15 = 24.$$

Finally, we assign the common sign to the sum. That is,

$$-9 + (-15) = -24.$$

2.2.2 Addition of Integers with different Signs

The Addition of Integers with Different Signs
To add Integers with different signs,
1) find the absolute value of each addend,
2) subtract the smaller absolute value from the larger absolute value, and then
3) write the result with the sign of the addend that had the larger absolute value.

EXAMPLE 1 Simplify the expression $11 + (-25)$.

SOLUTION: The absolute value of each number is

$$|11| = 11, \text{ and } |-25| = -(-25) = 25.$$

Subtracting the smaller absolute value from the larger absolute gives

$$25 - 11 = 14.$$

Since the larger absolute value belonged to the negative term, we have that $11 + (-25) = -14$.

EXAMPLE 2 Simplify the expression $-48+79$.

SOLUTION: The absolute value of each number is

$$|-48| = -(-48) = 48$$

and

$$|79| = 79.$$

Subtracting the smaller absolute value from the larger absolute gives

$$79-48 = 31.$$

Since the larger absolute value belonged to the positive term, we have that

$$-48+79 = 31.$$

EXAMPLE 3 Simplify the expression $-4,598+2,563$.

SOLUTION: The absolute value of each number is

$$|-4,598| = -(-4,598) = 4,598 \text{ and } |2,563| = 2,563.$$

Subtracting the smaller absolute value from the larger absolute value gives us

$$\begin{array}{r} 4\ 5\ 9\ 8 \\ -\ 2\ 5\ 6\ 3 \\ \hline 2\ 0\ 3\ 5 \end{array}$$

Since the larger absolute value belonged to the negative term, $-4,598+2,563 = -2,035$.

2.2.3 Subtraction of Integers

The subtraction of two integers can be thought of as adding the opposite. That is,

$$24 - 76$$

can be thought of as adding the opposite of 76 to 24. That is,

$$24 - 76 = 24 + (-76).$$

Thinking of subtraction in this way allows us to use the ideas from the previous examples.

Quick Note

To find the difference, $a - b$, we add a and the opposite of b.

$$a - b = a + (-b)$$

EXAMPLE 1 Simplify the expression $-17 - 12$.

SOLUTION: Here we are subtracting 12 from -17 so we can view this as adding the opposite of 12 to -17. That is,

$$-17 - 12 = -17 + (-12).$$

To find this sum we follow the rule for adding two integers with the same sign. So,

$$-17 - 12 = -17 + (-12) = -29.$$

EXAMPLE 2 Simplify the expression $-9 - 31$.

SOLUTION: Here we are subtracting 31 from -9 so we can view this as adding the opposite of 31 to -9. That is,

$$-9 - 31 = -9 + (-31).$$

To find this sum we follow the rule for adding two integers with the same sign. So,

$$-9 - 31 = -9 + (-31) = -40.$$

The next few examples illustrate how to deal with subtracting a negative number.

EXAMPLE 3 Simplify the expression $5-(-12)$.

SOLUTION: Here we are subtracting -12 from 5 so we can view this as adding the opposite of -12 to 5. That is,

$$5-(-12)=5+12$$

since the opposite of -12 is $-(-12)=12$. So,

$$5-(-12)=5+12=17.$$

EXAMPLE 4 Simplify the expression $-17-(-38)$.

SOLUTION: Here we are subtracting -38 from -17 so we can view this as adding the opposite of -38 to -17. That is,

$$-17-(-38)=-17+38$$

since the opposite of -38 is $-(-38)=38$.So,

$$-17-(-38)=-17+38=21.$$

Helpful Hint: Subtracting a Negative

Notice in the above examples that when we subtract a negative number, we are really adding that number. This is always true. That is, for a and b integers with $b \geq 0$,

$$a-(-b)=a+b.$$

Many like to remember this as "minus a negative" is "plus."

This idea that subtracting a negative becomes addition is explored in greater depth in the next section.

2.2.4 Order of Operations

The Order of Operations learned in Chapter 1 apply here and in all future calculations. One new addition would be handling the subtraction of a negative.

Caution

Your calculator has a key for subtraction and a key for negation. If you use the negation key for subtraction your calculator will produce an error. The subtraction key is used between two numbers while the negation key is used in front of a number.

The negation key: $\boxed{(-)}$

The subtraction key: $\boxed{-}$

EXAMPLE 1 Simplify the expression $-12-(-15)+4$.

SOLUTION: Performing addition and subtraction as they occur from left to right gives us the following.

$$-12-(-15)+4 = -12-(-15)+4$$

$$= -12+15+4$$

$$= 3+4$$

$$= 7.$$

EXAMPLE 2 Simplify the expression $13+(-29)-(-13)-7$.

SOLUTION: Performing addition and subtraction from left to right we get

$$13+(-29)-(-13)-7 = 13+(-29)-(-13)-7$$

$$= -16-(-13)-7$$

$$= -16+13-7$$

$$= -3-7$$

So, $13+(-29)-(-13)-7 = -10$.

2.2.5 Applications

EXAMPLE 1 John has \$250 in his bank account. He writes checks for \$185 and \$90. What is John's balance? Does he still have money in his account or is he overdrawn?

SOLUTION: Since each check decreases the account balance, John will have

$$\$250 - \$185 - \$90$$

in his account. Simplifying gives

$$\$250 - \$185 - \$90 = -\$25.$$

Because the balance is negative, John has overdrawn his account.

EXAMPLE 2 During a winter day in Minnesota, the temperature at 1:00 pm was 28°. Over the next six hours the temperature decreased 40°. What was the temperature at 7:00 pm that day?

SOLUTION: Because the temperature started at 28° and then decreased by 40°, we need to subtract 40° from 28°.

That is, the temperate six hours after 1:00 pm, which is 7:00 pm, would have been

$$28^\circ - 40^\circ = -12^\circ.$$

Section 2.2 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 8, add the integers.

1) $5+(-2)$ 2) $(-3)+(-4)$ 3) $(-7)+1$

4) $3+5$ 5) $-45+(-33)$ 6) $-129+367$

7) $-3,129+(-1,788)$ 8) $546+780$

For problems 9 - 17, subtract the integers.

9) $5-12$ 10) $-6-9$ 11) $14-8$

12) $8-14$ 13) $-38-73$ 14) $20-(-10)$

15) $-61-(13)$ 16) $1,230-(-875)$ 17) $-23-(-583)$

For problems 18 - 22, simplify the expression.

18) $11-(-6)+5-2$ 19) $-8+6+(-4)$ 20) $72+(-15)+(-8)+9$

21) $3+(-41)-(-12)-5$ 22) $14-13+(-2)-(-5)-3$

For problems 23 - 25, translate the English phrase into a mathematical expression and simplify.

23) The sum of negative 47, 62, and negative 3.

24) Five more than negative 18.

25) Negative 21 increased by 7.

26) The temperature is 45° F at 5 : 00 AM. By noon the temperature decreases 20° F. What is the temperature at noon?

27) The temperature is negative 10°F at 5 : 00 AM. Over the next 10 hours the temperature increases 25°F. What is the temperature at 3 : 00PM?

28) Mary has $300 in her checking account. During the week she makes a deposit of $40 and writes a check for $65. What is her balance at the end of the week.

Review.

29) Round $32,568$ to the hundred's place.

30) Write $8,392$ in expanded form.

31) Round 412 to the ten's place.

32) True or False: $-|-12| = 12$

33) True or False: $-45 < -72$

34) True or False: $|3| > |-5|$

35) Simplify: $2^3(7-4)+15 \div 3$

Chapter 2
Section 2.2 – Addition and Subtraction of Integers

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

1) _____

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35) _____

2.3 Multiplication and Division of Integers

2.3.1 Multiplication of Integers

We saw in Chapter 1 that multiplication can be thought of as repeated addition. For example, we said

$$3 \cdot 5 = 5+5+5 \quad \text{or} \quad 3 \cdot 5 = 3+3+3+3+3.$$

Either way we get that $3 \cdot 5 = 15$. Let's see what happens when we use this idea and start putting in negative numbers.

To see the pattern that will emerge, start by multiplying 3 by 5, and then by smaller integers allowing them to decrease to negative integers.

1) $3 \cdot 5 = 3+3+3+3+3 = 15$

2) $3 \cdot 4 = 3+3+3+3 = 12 = 15-3$

3) $3 \cdot 3 = 3+3+3 = 9 = 12-3$

4) $3 \cdot 2 = 3+3 = 6 = 9-3$

5) $3 \cdot 1 = 3 = 6-3$

Keep this pattern going. The numbers in red are decreasing by one every time which makes the numbers in blue decrease by three every time.

6) $3 \cdot 0 = 0 = 3+(-3)$

7) $3 \cdot -1 = -3 = 0+(-3)$

8) $3 \cdot -2 = (-3)+(-3) = -6 = (-3)+(-3)$

9) $3 \cdot -3 = (-3)+(-3)+(-3) = -9 = (-6)+(-3)$

10) $3 \cdot -4 = (-3)+(-3)+(-3)+(-3) = -12 = (-9)+(-3)$

What we need to recognize from this pattern is that a positive number times a positive number results in a positive number, while a positive number times a negative number results in a negative number.

We can use this same idea to determine the result of multiplying a negative number with a negative number.

1) $3 \cdot -4 = (-4) + (-4) + (-4) = -12$

2) $2 \cdot -4 = (-4) + (-4) = -8 = (-12) + 4$

3) $1 \cdot -4 = -4 = (-8) + 4$

4) $0 \cdot -4 = 0 = -4 + 4$

5) $-1 \cdot -4 = 4 = 0 + 4$

6) $-2 \cdot -4 = 8 = 4 + 4$

7) $-3 \cdot -4 = 12 = 8 + 4$

8) $-4 \cdot -4 = 16 = 12 + 4$

From this we see that a negative number times a negative number results in a positive number.

Moreover, we can see that an integer times zero is always zero.

From these patterns we have the following rules to use when multiplying integers.

Sign Rules for Integer Multiplication
Positive times positive results in a positive.
Positive times negative results in a negative.
Negative times negative results in a positive.
Any integer times zero results in zero.

Recall from Section 2.1 that we defined the opposite of a number a as $-a$. Being able to use integers rather than just whole numbers allows us to make this definition stronger in that we can say

$$\text{the opposite of a number } a = -1(a).$$

That is, the "opposite" of a number can be defined as negative one times that number.

Moreover, subtraction can be defined similarly. That is, for two integers a and b,

$$a - b = a + (-1)(b).$$

Quick Note

We typically use parentheses to indicate multiplication when negative numbers are being used.

EXAMPLE 1 Simplify the expression $7(-6)$.

SOLUTION: Since this is a product of a positive number and a negative number, the result will be a negative number. Furthermore, $7(6) = 42$ so

$$7(-6) = -42.$$

EXAMPLE 2 Calculate the product of -76 and -53.

SOLUTION: Since this is a product of two negative numbers the result will be positive. Furthermore we know that

$$\begin{array}{r} 7\ 6 \\ \times\ 5\ 3 \\ \hline 2\ 2\ 8 \\ +\ 3\ 8\ 0\ 0 \\ \hline 4\ 0\ 2\ 8 \end{array}$$

Therefore, $(-76)(-53) = 4,028$.

2.3.2 Division of Integers

Just as in previous sections we can use knowledge of multiplication to evaluate the division of two integers. That is, we know that

$$3(4) = 12$$

so we can say that $\dfrac{12}{4} = 3$.

Similarly, since we know that $3(-4) = -12$ we are able to say that

$$\frac{-12}{-4} = 3, \text{ and that } \frac{-12}{3} = -4.$$

Notice also that $(-3)(-4) = 12$ so we can say

$$\frac{12}{-3} = -4.$$

Putting these together provides the sign rules for division.

Sign Rules for Integer Division
Positive divided by positive results in a positive.
Positive divided by negative results in a negative.
Negative divided by negative results in a positive.
Any integer divided by zero is undefined.

Note that the first three are identical to the sign rules for Integer Multiplication with "times" replaced with "divided by." The last rule is an extension of the rule from Section 1.2.6.

In fact, throughout your studies in mathematics, remembering that

"*any number times zero is zero*" and "*any number divided by zero is undefined*"

will help you avoid many pitfalls.

EXAMPLE 1 Evaluate $72 \div (-9)$.

SOLUTION: Since this is the quotient of a positive number and a negative number, the result will be negative. Moreover since

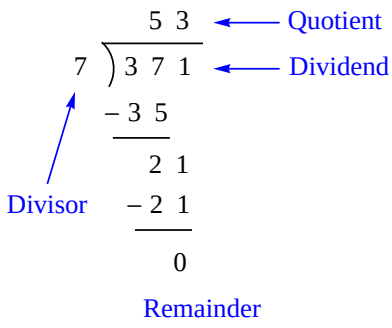
$$\frac{72}{9} = 8,$$

we have that $\frac{72}{-9} = -8$.

EXAMPLE 2 Evaluate $-371 \div (-7)$.

SOLUTION: Since this is the quotient of a negative number and a negative number, the result will be positive. Moreover, long division shows that

$$\frac{371}{7} = 53.$$



EXAMPLE 3 Evaluate $0 \div (-15)$.

SOLUTION: in Section 1.2.6 we showed that zero divided by any non-zero whole number is zero. This holds true for integers as well. Therefore,

$$0 \div (-15) = 0.$$

EXAMPLE 4 Evaluate $-8 \div 0$.

SOLUTION: in Section 1.2.6 we showed that any non-zero whole number divided by zero is undefined. This holds true for integers as well. Therefore,

$$-8 \div 0 \text{ is undefined.}$$

2.3.3 Natural Number Exponents

Recall from Section 1.2.3 that the operation of multiplication really represents repeated addition. That is, the expression

$$2+2+2+2+2+2+2 \text{ can be written as } 2 \cdot 7.$$

We then went on to represent repeated multiplication using notation called **Exponents**. As an example, the expression

$$3 \cdot 3 \cdot 3 \cdot 3 \text{ can be written as } 3^4.$$

In the expression 3^4, the number 3 is called the "base" and the number 4 is called the "exponent."

Natural Number Exponents

Given any whole number a and any natural number n:

$$a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a \cdot a}_{\text{multiply the base } a, n \text{ times}}.$$

We call the number a the **base** and n the **exponent**.

EXAMPLE 1 Evaluate $(-2)^3$.

SOLUTION: Be definition, $(-2)^3$ is -2 multiplied by itself three times. That is,

$$(-2)^3 = (-2)(-2)(-2).$$

Performing the multiplication from left to right we get

$$(-2)^3 = (-2)(-2)(-2) = 4(-2) = -8.$$

EXAMPLE 2 Evaluate $(-5)^2$.

SOLUTION: Be definition, $(-5)^2$ is -5 multiplied by itself two times. That is,

$$(-5)^2 = (-5)(-5) = 25.$$

EXAMPLE 3 Evaluate $(-3)^4$.

SOLUTION: Be definition, $(-3)^4$ is -3 multiplied by itself four times. That is,

$$(-3)^4 = (-3)(-3)(-3)(-3) = 81.$$

Make the observation that any negative number raised to an even natural number is positive and that any negative number raised to an odd power is negative. This result follows from the product of two negative numbers is positive and the product of a positive and a negative is negative.
With that said, it is important to identify what the base is in an expression involving exponents.

EXAMPLE 4 Compare $(-6)^2$ and -6^2.

SOLUTION: The difference between the expression $(-6)^2$ and -6^2 is in the use of the parentheses. For the expression $(-6)^2$, the base is -6 while in the expression -6^2, the base is 6. That is, because the -6 is in parentheses in the expression $(-6)^2$, the exponent of 2 is acting on it. The expression -6^2 really means the opposite of 6^2.
This means that

$$(-6)^2 = (-6)(-6) = 36 \text{ while } -6^2 = -(6)(6) = -36.$$

2.3.4 Order of Operations

Quick Note

Parentheses and brackets are commonly used grouping symbols. In future sections the numerators and denominators of fractions as well as absolute values and radicals will be grouping symbols.

Order of Operations

1) Perform all operations inside parentheses, or any grouping symbols, first.

2) Simplify any expressions containing exponents.

3) Perform multiplication and division as they occur from left to right.

4) Perform addition and subtraction as they occur from left to right.

Notice that the Order of Operations has not changed. The only difference between the expressions we are simplifying now and the expressions we simplified in earlier sections is that we now allow for negative answers.

EXAMPLE 1 Simplify $2-4(3-8)^2$.

SOLUTION: Following the Order of Operations we get the following.

$$2-4(3-8)^2 = 2-4(-5)^2$$

$$= 2-4(25)$$

$$= 2-100$$

$$= -98.$$

Therefore, $2-4(3-8)^2 = -98$.

One important thing to remind yourself of when working on simplifying expressions is that every problem will follow the same rules. This means that regardless of how "complicated" an expression may appear, the steps you need to take in order to simplify it are always the same. This is one of the main reasons knowing the Order of Operations is so crucial. The Order of Operations is universal. Anyone wishing to **correctly** simplify any mathematical expression must follow them.

EXAMPLE 2 Simplify $6[14-3(4-2\cdot 3)^2]\div(-3)$.

SOLUTION: Using the Order of Operations,

$$6[14-3(4-2\cdot 3)^2]\div(-3)$$

$$=6[14-3(4-6)^2]\div(-3)$$

$$=6[14-3(-2)^2]\div(-3)$$

$$=6[14-3(4)]\div(-3)$$

$$=6[14-12]\div(-3)$$

$$=6[2]\div(-3)$$

$$=12\div(-3)$$

$$=-4.$$

Therefore, $6[14-3(4-2\cdot 3)^2]\div(-3)=-4$.

EXAMPLE 3 Simplify $[(-6+2)^3+7]\div[-5-(-2)]$.

SOLUTION: With the Order of Operations we get

$$[(-6+2)^3+7]\div[-5-(-2)]$$

$$=[(-6+2)^3+7]\div[-5-(-2)]=[(-4)^3+7]\div[-5+2]$$

$$=[-64+7]\div[-3]$$

$$=[-57]\div[-3]$$

$$=19.$$

Helpful Hint: Things to Remember

Subtracting Negatives

Subtracting a number is the same as adding its opposite: $a - b = a + (-b)$.

Natural Number Exponents

Given any integer a and any natural number n, a^n means a multiplied by itself n times.

Order of Operations

1) Perform all operations inside parentheses, or any grouping symbols, first.

2) Simplify any expressions containing exponents.

3) Perform multiplication and division as they occur from left to right.

4) Perform addition and subtraction as they occur from left to right.

Sign Rules

Positive multiplied or divided by a positive is positive.

Positive multiplied or divided by a negative is negative.

Negative multiplied or divided by a negative is positive.

Any number multiplied by zero is zero.

Any number divided by zero is undefined.

Section 2.3 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 7, perform the indicated multiplication.

1) $(-2)(-5)$ 2) $0(-4)$ 3) $8(-3)$

4) $(-5)(-3)$ 5) $3(6)$ 6) $-7(2)(-4)$

7) $-2(-3)(-1)$ 8) $-47(19)$ 9) $-120(-53)$

For problems 10 - 12, evaluate the expression.

10) $(-9)^2$ 11) -9^2 12) $(-4)^3$

13) -4^3 14) -3^4 15) $(-3)^4$

For problems 13 - 18, perform the division.

16) $64 \div (-8)$ 17) $\dfrac{-120}{60}$ 18) $(-36) \div (-18)$

19) $0 \div (-12)$ 20) $\dfrac{-399}{21}$ 21) $25 \div 0$

22) $-61 \div 0$ 23) $(-24) \div (-8) \div (3)$ 24) $(-465) \div (-15)$

25) $0 \div (-2) \div 8$

For problems 26 - 28, simplify the expression.

26) $-3[5-(7-9)^3+(-9)]^2+14 \div (-7)$ 27) $[-(5+(-6))^5+7] \div (12 \div 3)$

28) $-2(4)(-7)-3(5)^2$

For problems 29 - 31, translate the English expression into a mathematical expression and simplify.

29) Multiply 47 and negative seventy.

30) The product of negative ten and negative seven.

31) The quotient of negative 12 and 3.

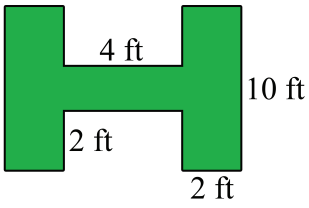
Review.

32) A Standard lot in Palm Coast has a width of 52 feet and a length of 192 feet. What is the total area of the lot?

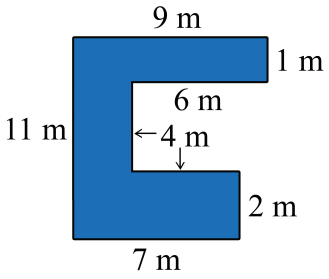
33) Peter wants to paint his fence. He has a privacy fence with the dimensions of 72 feet by 6 feet. What is the area of his fence?

Find the perimeter.

34)



35)



Chapter 2
Section 2.3 – Multiplication and Division of Integers

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

1) _____

2) _____

3) _____

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6) _____

7) _____

8) _____

9) _____

10) _____

11) _____

12) _____

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INTRODUCTION TO RATIONAL NUMBERS

CHAPTER 3

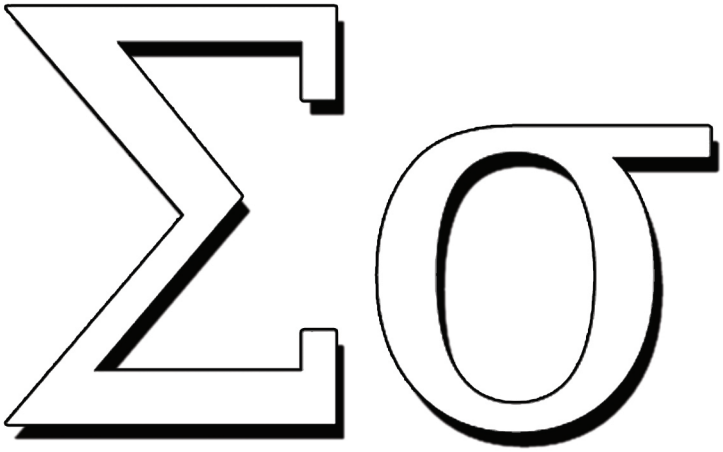
CHAPTER OBJECTIVES

3.1 Introduction to Rational Numbers: Fractions and Mixed Numbers

3.2 Prime and Composite Numbers: Factorizations

3.3 Simplifying Fractions and Equivalent Fractions

The Sigma Symbol



Sigma is the 18th letter of the Greek alphabet and is equivalent to our letter 'S'. In mathematics, the upper case sigma is used for the summation notation. The lower case sigma stands for standard deviation. Each has its own unique formula, and yes, both the upper case and lower case look vastly different from each other. Neither of them looks like our letter 'S', but they are both the Greek equivalent of it. If you notice, the two formulas that use these two symbols both start with the letter 's'. The upper case sigma is used in the summation notation. This particular notation is called sigma notation. This math symbol was first used by Leonhard Euler, a Swiss mathematician, in representing the addition of a series of mathematical expressions. The lower case sigma is used for the standard deviation formula in statistics. This particular notation is also called sigma notation. Standard deviation is a measure of the dispersion of a set of data from its mean. If the data points are further from the mean, there is higher deviation within the data set. Standard deviation is calculated as the square root of variance by determining the variation between each data point relative to the mean.

3.1 Introduction to Rational Numbers: Fractions and Mixed Numbers

In this section we begin investigating the use of **Rational Numbers**, denoted as $\mathbb{Q}$.

Rational Numbers
A Rational Number, or fraction, is any number of the form
$\frac{a}{b}$ where a and b are Integers with $b \neq 0$. The "top" of the fraction a is referred to as the **Numerator**. The "bottom" of the fraction b is referred to as the **Denominator**.

We have worked with the notation $\frac{a}{b}$ in Sections 1.2 and 2.3 when dividing Whole Numbers and Integers. That is to say, that when we wrote the "fraction"

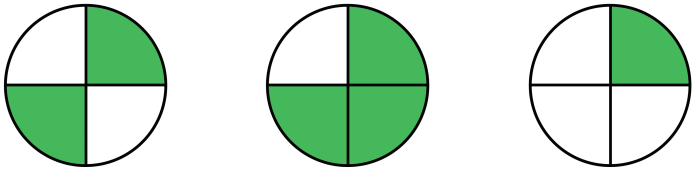
$$\frac{24}{4}$$

we meant $24 \div 4$ which simplifies to 6. Moreover, the set of Whole Numbers and Integers are considered to be Rational Numbers since for any integer a,

$$a = a \div 1 = \frac{a}{1}.$$

One large difference between the Rational Numbers that we worked with in Chapters 1 and 2 and the Rational Numbers that we are adding in here is that when we performed the division with Integers, the remainders were always zero. All this means is that in Chapter 1 when we divided two Whole Numbers, we got a Whole Number. In Chapter 2, when we divided two Integers, we got an Integer. The extension that we make now is that we are considering those fractions that represent parts of wholes.

We can visualize fractions representing "parts of wholes" as follows.



On the left, two of the four sections of the circle are shaded. This can be represented by the fraction $\frac{2}{4}$. In the middle, three of the four sections are shaded. This can be represented by the fraction $\frac{3}{4}$. The figure on the right can be represented by the fraction $\frac{1}{4}$.

EXAMPLE 1 Given the fraction $\frac{2}{5}$, identify the numerator and denominator, and write the fraction using division.

SOLUTION: The numerator of the fraction is 2 and the denominator is 5.

Using division, $\frac{2}{5}$ is equivalent to $2 \div 5$.

EXAMPLE 2 Given the fraction $\frac{15}{3}$, identify the numerator and denominator, and write the fraction using division.

SOLUTION: The numerator of the fraction is 15 and the denominator is 3.

Using division, $\frac{15}{3}$ is equivalent to $15 \div 3$. Since $15 \div 3 = 5$,

$$\frac{15}{3} = 15 \div 3 = 5.$$

All Rational Numbers have a numerator and a denominator. They can be classified as either **Proper** or **Improper** fractions based on the "size" of the numerator compared to the "size" of the denominator. We say that a fraction is a proper fraction if the numerator is smaller than the denominator and we say that a fraction is an improper fraction if the numerator is larger than or equal to the denominator. It is important to remember that the use of the words proper and improper have no connection at all with "good" or "bad;" they simply refer to the size of the numerator as compared to the size of the denominator.

Proper Fractions
The fraction $\frac{a}{b}$, a and b Integers with $b \neq 0$ is a **proper** fraction if $a < b$.
Improper Fractions
The fraction $\frac{a}{b}$, a and b Integers with $b \neq 0$ is an **improper** fraction if $a \geq b$.

EXAMPLE 3 Identify the fractions $\frac{5}{7}$, $\frac{12}{11}$, and $\frac{5}{5}$ as either proper or improper.

SOLUTION: The fraction

$$\frac{5}{7}$$

is a proper fraction because the numerator is less than the denominator. That is, because $5 < 7$, $\frac{5}{7}$ is a proper fraction.

For both $\frac{12}{11}$ and $\frac{5}{5}$, the numerator $\geq$ the denominator. That is, because

$$12 \geq 11, \text{ and } 5 \geq 5,$$

both $\frac{12}{11}$ and $\frac{5}{5}$ are improper fractions.

3.1.1 Mixed Numbers

A **Mixed Number** is a number that is a combination of an Integer and a non-integer Rational Number. We will discuss how to convert from an improper fraction to a mixed number and back again.

Some examples of mixed numbers are

$$3\frac{1}{4},\ -2\frac{6}{11},\ \text{and}\ 1\frac{3}{5}.$$

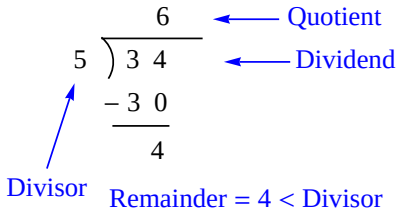
The mixed number $3\frac{1}{4}$ is read as "three and one fourth."

Converting Improper Fractions to Mixed Numbers

As mentioned earlier, the Rational Numbers we worked with in Chapters 1 and 2 were Whole Numbers and Integers. When we performed division, the remainder was always zero. This ensured that when we divided two Whole Numbers, or two Integers, we got back a Whole Number, or an Integer.

In general, this is not always the case. In fact, typically there will be a non-zero remainder after performing division.

Consider the mixed number $\frac{34}{5}$. Because this is equivalent to saying $34 \div 5$, let's perform the division.



1. 5 goes into 34 evenly 6 times
2. Multiply $6 \cdot 5 = 30$
3. Subtract $34 - 30 = 4$
4. The remainder is less than the divisor so we are done.

The integer part of the mixed number will be the quotient $= 6$ and the fraction part will be

$$\frac{\text{remainder}}{\text{divisor}} = \frac{4}{5},$$

therefore, $\frac{34}{5} = 6\frac{4}{5}$.

When we read a mixed number, such as $6\frac{4}{5}$, we read it as

$$6\frac{4}{5} = \text{six and four fifths.}$$

The mixed number itself really means $6+\frac{4}{5}$. We will investigate how to add fractions in the next chapter. At the moment we simply wish to become adjusted to working with non-integer rational numbers.

EXAMPLE 1 Write the improper fraction $-\frac{235}{3}$ as a mixed number.

SOLUTION: Ignoring the sign and performing the division $235 \div 3$ gives

$$\begin{array}{r} 78 \\ 3\overline{)235} \\ -21 \\ \hline 25 \\ -24 \\ \hline 1 \end{array}$$

78 ← Quotient; 235 ← Dividend; 3 ← Divisor

Remainder = 1 < Divisor

Therefore, $-\frac{235}{3} = -78\frac{1}{3}$.

Converting Mixed Numbers to Improper Fractions

As mentioned previously, a mixed number is really the sum of an integer and a non-integer Rational Number. As we do not discuss the addition of fractions until Chapter 4, we will discuss an algorithm that can be used to convert a mixed number to an improper fraction.

Keep in the back of your mind that this is also the method of adding an integer and a non-integer fraction, and by extension, the method to add two non-integer fractions.

Converting Mixed Numbers to Improper Fractions

To convert the mixed number

$$a\frac{b}{c},$$

to an improper fraction,

1) calculate $a(c)+b$,

2) form the improper fraction $\dfrac{a(c)+b}{c}$

Quick Note

Unless you are told specifically to give an answer as a mixed number, leave answers as improper fractions. In future sections improper fractions are preferred.

EXAMPLE 2 Write the mixed number $5\frac{3}{7}$ as an improper fraction.

SOLUTION: Following the algorithm given above with

$$a=5,\ b=3,\ \text{and}\ c=7$$

we get

$$5\frac{3}{7}=\frac{5(7)+3}{7}=\frac{38}{7}.$$

EXAMPLE 3 Write the mixed number $-6\frac{7}{12}$ as an improper fraction.

SOLUTION: Ignoring the negative sign for the moment and following the algorithm given above with

$$a=6,\ b=7,\ \text{and}\ c=12$$

we get

$$6\frac{7}{12}=\frac{6(12)+7}{12}=\frac{79}{12}$$

Because the original mixed number was negative we get $-\dfrac{79}{12}$.

Section 3.1 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 3, identify the numerator and denominator.

1) $\frac{1}{4}$ 2) $\frac{3}{7}$ 3) $\frac{9}{5}$

For problems 4 - 7, write the fraction as a division problem and simplify if possible.

4) $\frac{8}{2}$ 5) $\frac{4}{4}$ 6) $\frac{0}{5}$ 7) $\frac{8}{0}$

For problems 8 - 13, identify the fraction as either proper or improper.

8) $\frac{3}{4}$ 9) $\frac{9}{7}$ 10) $\frac{20}{20}$ 11) $\frac{23}{20}$

12) $-\frac{1}{3}$ 13) $-\frac{0}{42}$

For problems 14 - 17, convert the improper fraction into a mixed number.

14) $\frac{22}{3}$ 15) $\frac{38}{5}$ 16) $\frac{125}{12}$ 17) $\frac{27}{13}$

For problems 18 - 21, convert the mixed number into an improper fraction.

18) $2\frac{3}{4}$ 19) $7\frac{1}{3}$ 20) $10\frac{5}{6}$ 21) $-12\frac{1}{6}$

For problems 22 - 24, write the quotient as a mixed number.

22) $7 \div 3$ 23) $16 \div 5$ 24) $-5 \div 2$

25) In a Daytona Beach neighborhood, there are 71 houses. 17 houses are for sale. Write a fraction that represents the portion of houses that are for sale.

Review. Simplify the expression.

26) $10+5$ 27) $10-5$ 28) $10+(-5)$ 29) 3×4

30) $-3(4)$ 31) $2(-6)$ 32) $5(-4)$ 33) $-2(-7)$

34) $\frac{123}{3}$ 35) $121 \div (-11)$

Chapter 3
Section 3.1 – Introduction to Rational Numbers: Fractions and Mixed Numbers

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

1) Numerator: _____ Denominator: _____

2) Numerator: _____ Denominator: _____

3) Numerator: _____ Denominator: _____

4) _____

5) _____

6) _____

7) _____

8) _____

9) _____

10) _____

11) _____

12) _____

13) _____

14) _____

15) _____

16) _____

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29) _____

30) _____

31) _____

32) _____

33) _____

34) _____

35) _____

3.2 Prime and Composite Numbers: Factorizations

A **Prime Number** is defined to be any Natural Number that has two divisors, namely, 1 and itself. A **Composite Number** is any Natural Number that can be written as the product of prime numbers. Note that the number 1 is neither prime nor composite. It is not prime since it does not have two divisors (the only divisor of 1 is 1). It is not composite because it cannot be written as the product of prime numbers.

Some prime numbers are

$$2, 3, 5, 7, 11, 13, 17, 19, 23, \ldots.$$

Note that 2 is the smallest prime number and the only even prime number.

3.2.1 Factorizations of a Number

Quick Note

How large can a prime number be? There actually isn't a "largest" prime number. In fact, a modern challenge among pure mathematicians and computer programmers is to find the largest prime number. At the time this book was written, the largest prime number is

$$2^{25,964,951} - 1.$$

This is a number with 7,816,230 digits.

Finding a factorization of a number means finding two or more factors that multiply to that number. As an example, a factorization of 6 is $2(3)$ so we say that 2 and 3 are "factors" of the number 6. Note that this factorization is not unique since we could have written 6 as $1(6)$.

In order to help determine the factors of a given number there are **divisibility tests**. The following is a short list of divisibility tests.

- *Divisible by* 2 - If the digit in the ones place is divisible by 2, the number is divisible by 2.
- *Divisible by* 3 - If the sum of the digits is divisible by 3, the number is divisible by 3.
- *Divisible by* 5 - If the digit in the ones place is 0 or 5, the number is divisible by 5.
- *Divisible by* 6 - If the number is divisible by 2 and 3, the number is divisible by 6.
- *Divisible by* 9 - If the sum of the digits is divisible by 9, the number is divisible by 3.
- *Divisible by* 10 - If the digit in the ones place is 0, the number is divisible by 10.

While there are many more divisibility tests, this short list can be memorized quickly and applied to any factorization problem. The next few examples demonstrate how to use these tests and a few others.

Make the note that the *Divisible by* 2 test is really a way to tell whether a given number is even since any number divisible by 2 must be even. This is actually why 2 is the only even prime number.

EXAMPLE 1 Find all of the different factorizations for 36.

SOLUTION: A **trivial factorization** of any number is 1 times the number. So one of the factorizations is

$$36 = 1 \cdot 36.$$

Since the ones place of 36 is divisible by 2, we know that 36 is divisible by 2. Therefore, since $36 \div 2 = 18$ another factorization is

$$36 = 2 \cdot 18.$$

If we add the digits of 36, we get $3 + 6 = 9$ and since 9 is divisible by 3, 36 is divisible by 3. Then, since $36 \div 3 = 12$ another factorization is

$$36 = 3 \cdot 12.$$

Because 36 is divisible by 2 and 3 it is divisible by $2(3) = 6$. Another factorization is

$$36 = 6 \cdot 6.$$

Notice that when we added the digits of 36 we got $3 + 6 = 9$. Because this is divisible by 9, 36 is divisible by 9. Another factorization is

$$36 = 4 \cdot 9.$$

Because 4 is even and can be written as $2 \cdot 2$, another factorization is

$$36 = 4 \cdot 9 = 2 \cdot 2 \cdot 9.$$

Because 6 is composite and can be factored as $2 \cdot 3$, another factorization is

$$36 = 6 \cdot 6 = 2 \cdot 3 \cdot 6.$$

Lastly, since $4 = 2 \cdot 2$ and $9 = 3 \cdot 3$, another factorization is

$$36 = 4 \cdot 9 = 2 \cdot 2 \cdot 3 \cdot 3.$$

Altogether there are 8 factorizations of the number 36.

EXAMPLE 2 Find all of the different factorizations for 45.

SOLUTION: The trivial factorization is

$$45 = 1 \cdot 45.$$

Because 5 is not divisible by 2 (5 is not even), we know that 45 is not divisible by 2.

The sum of the digits is $4 + 5 = 9$ so we know that 45 is divisible by 3 and 9. Therefore,

$$45 = 3 \cdot 15 \text{ and } 45 = 9 \cdot 5.$$

Because 15 can be factored as $3 \cdot 5$, another factorization is

$$45 = 3 \cdot 15 = 3 \cdot 3 \cdot 5.$$

As these are the only ways to factor 45, we are done.

Observe that the previous two examples demonstrate that there is not always a unique factorization of a number. However, there is always one unique way to factor a number as the product of prime factors.

3.2.2 Prime Factorization

The prime factorization of a number is found by writing the number as a product of prime numbers. From the last two examples we can see that the prime factorization of 36 and 45 are

$$36 = 2 \cdot 2 \cdot 3 \cdot 3 = 2^2 \cdot 3^2 \quad \text{and} \quad 45 = 3 \cdot 3 \cdot 5 = 3^2 \cdot 5.$$

EXAMPLE 1 Find the prime factorization of 24.

SOLUTION: Since 24 is even we know that it is divisible by 2. Then, since $24 \div 2 = 12$ we know that

$$24 = 2 \cdot 12.$$

We know this is not the prime factorization of 24 since the factor 12 is not prime. However, since 12 is even it is divisible by 2. More specifically,

$$12 = 2 \cdot 6$$

and so

$$24 = 2 \cdot 12 = 2 \cdot 2 \cdot 6.$$

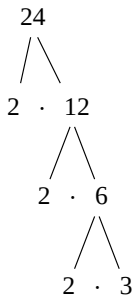
Again we see that this is not the prime factorization since 6 is even and $6 = 2 \cdot 3$. Therefore,

$$24 = 2 \cdot 2 \cdot 6 = 2 \cdot 2 \cdot 2 \cdot 3.$$

Now since every factor is prime we are done. The prime factorization of 24 is

$$24 = 2^3 \cdot 3.$$

Many times it is convenient to keep track of the factoring process using what's called a factor tree. The factor tree for this problem is shown here.



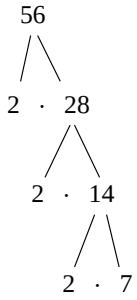
Avoiding Mistakes

Make sure that the end of each branch is a prime number.

Note that every step in the above explanation is a line in the factor tree.

EXAMPLE 2 Find the prime factorization of 56.

SOLUTION: For this problem the factor tree can be written as



Quick Note

Finding the prime factorization of numbers will be important later when adding unlike fractions.

Note that each line of the factor tree follows since

$$56 \div 2 = 28 \text{ so } 56 = 2 \cdot 28$$

$$28 \div 2 = 14 \text{ so } 28 = 2 \cdot 14$$

$$14 \div 2 = 7 \text{ so } 14 = 2 \cdot 7$$

Because both 2 and 7 are prime numbers we are done and $56 = 2^3 \cdot 7$.

EXAMPLE 3 Find the prime factorization of 105.

SOLUTION: Note that 105 is not even so it is not divisible by 2. However, since the sum of its digits $1+0+5=6$, is divisible by 3, we know that 105 is divisible by 3.

$$105 = 3 \cdot 35.$$

For 35, the ones digit is 5 so we know it is divisible by 5.

$$35 = 5 \cdot 7$$

and so

$$105 = 3 \cdot 35 = 3 \cdot 5 \cdot 7.$$

Since every factor is prime, the prime factorization of $105 = 3 \cdot 5 \cdot 7$.

Section 3.2 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 6, find two different factorizations for the given number.

1) 12 2) 16 3) 42 4) 15

5) 36 6) 72

For problems 7 - 10, determine if the given number is divisible by 2, 3, 5, 9, and 10.

7) 123 8) 333 9) 125 10) 3330

For problems 11 - 18, determine if the given number is prime, composite, or neither.

11) 0 12) 1 13) 2 14) 3

15) 12 16) 19 17) 57 18) 121

For problems 19 - 20, determine if the given factorization is a prime factorization.

19) $48 = 2 \cdot 3 \cdot 8$ 20) $210 = 2 \cdot 3 \cdot 5 \cdot 7$

For problems 21 - 24, find the prime factorization of the given number.

21) 57 22) 60 23) 105 24) 128

For problems 25 - 27, list all of the factors of the given number.

25) 64 26) 91 27) 360

Review. Simplify the expression.

28) $15 + (-5)$ 29) $15 - (-5)$ 30) $0 \cdot 12,345$ 31) $1 \cdot 12,345$

32) $0 \div 1$ 33) $1 \div 0$ 34) $36 \div (-12)$

35) Convert $\frac{57}{5}$ to a mixed number.

Chapter 3
Section 3.2 – Prime and Composite Numbers: Factorizations

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

1) _____

2) _____

3) _____

4) _____

5) _____

6) _____

7) _____

8) _____

9) _____

10) _____

11) _____

12) _____

13) _____

14) _____

15) _____

16) _____

17) _____

18) _____

19) _____

20) _____

21) _____

22) _____

23) _____

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25) _____

26) _____

27) _____

28) _____

29) _____

30) _____

31) _____

32) _____

33) _____

34) _____

35) _____

3.3 Simplifying Fractions and Equivalent Fractions

3.3.1 Simplifying Fractions

A fraction is said to be "simplified", in "simplified form", or "reduced" when the numerator and denominator do not share any common factors. To determine whether or not the numerator and denominator share common factors we will use the factorization techniques from the previous section.

The process of simplifying a fraction is done by identifying factors common to both the numerator and denominator and then eliminating them from both. To find common factors between the numerator and denominator we could

1) find all of the factors of the numerator and of the denominator and identify the ones they have in common,
2) write the prime factorization of the numerator and denominator and identify the prime factors they have in common,
3) or identify the greatest common factor of the numerator and denominator.

Regardless of the approach you take in simplifying a fraction, always provide fractions in simplified (reduced) form.

EXAMPLE 1 Simplify the fraction $\frac{8}{20}$.

SOLUTION: One way to simplify fractions is to find all of the factors of the numerator and of the denominator.

The factors of 8 are 1, 2, 4, and 8. The factors of 20 are 1, 2, 4, 5, and 20.

We see that both the numerator and denominator share a common factor of 4. If we eliminate the common factor we see that

$$\frac{8}{20} = \frac{2 \cdot 4}{5 \cdot 4} = \frac{2 \cdot \cancel{4}}{5 \cdot \cancel{4}} = \frac{2}{5}.$$

Therefore, $\frac{8}{20}$ simplifies to $\frac{2}{5}$.

Avoiding Mistakes

Simplifying fractions can be done vertically and diagonally but not horizontally.

Notice that it is not important to identify all of the factors of the numerator and denominator, only the factors that they share in common.

EXAMPLE 2 Simplify the fraction $\frac{75}{100}$.

SOLUTION: To simplify this fraction we will write out the prime factorization of the numerator and denominator. That is,

$$75 = 5 \cdot 15 = 5 \cdot 3 \cdot 5 = 3 \cdot 5^2$$

and

$$100 = 2 \cdot 50 = 2 \cdot 2 \cdot 25 = 2 \cdot 2 \cdot 5 \cdot 5 = 2^2 \cdot 5^2.$$

Avoiding Mistakes

The fraction $\frac{a}{b}$ can be written as a/b, however, we discourage the use the "slanted" fraction bar as it's use may cause confusion in later applications.

Note that both the numerator and denominator share a common factor of $5^2 = 25$. That is,

$$\frac{75}{100} = \frac{3 \cdot 5^2}{2^2 \cdot 5^2}.$$

Eliminating the common factor we get

$$\frac{75}{100} = \frac{3 \cdot \cancel{5^2}}{2^2 \cdot \cancel{5^2}} = \frac{3}{2^2} = \frac{3}{4}.$$

Therefore, $\frac{75}{100}$ simplifies to $\frac{3}{4}$.

Greatest Common Factor

Each of the methods shown in examples 1 and 2 lead to finding what is called the **Greatest Common Factor** of the numerator and denominator. The Greatest Common Factor of two numbers is precisely the largest factor common to both numbers.

In fact, from example 1, we see that the Greatest Common Factor of 8 and 20 is 4. From example 2, we see that the Greatest Common Factor of 75 and 100 is 25.

The method of identifying and eliminating the Greatest Common Factor from the numerator and denominator is the preferred method used to simplify fractions.

EXAMPLE 3 Simplify the fraction $\frac{60}{126}$.

SOLUTION: The prime factorization for the numerator and denominator are

$$60 = 2^2 \cdot 3 \cdot 5 \quad \text{and} \quad 126 = 2 \cdot 3^2 \cdot 7.$$

From this we see that both the numerator and denominator share the factors 2 and 3. Therefore, the Greatest Common Factor of 60 and 126 is 6 and

$$\frac{60}{126} = \frac{2^2 \cdot 3 \cdot 5}{2 \cdot 3^2 \cdot 7} = \frac{6 \cdot 10}{6 \cdot 21} = \frac{\cancel{6} \cdot 10}{\cancel{6} \cdot 21} = \frac{10}{21}.$$

So $\frac{60}{126}$ simplifies to $\frac{10}{21}$.

Using prime factorizations to find the Greatest Common Factor will always work however, there will be many times that simple inspection will be more efficient.

EXAMPLE 4 Simplify the fraction $\frac{36}{72}$.

SOLUTION: Note that both the numerator and denominator share a common factor of 36. That is,

$$\frac{36}{72} = \frac{1 \cdot 36}{2 \cdot 36}.$$

Eliminating the common factor we get

$$\frac{36}{72} = \frac{1 \cdot \cancel{36}}{2 \cdot \cancel{72}} = \frac{1}{2}.$$

Therefore, $\frac{36}{72}$ simplifies to $\frac{1}{2}$.

Avoiding Mistakes

Don't forget to write the "1" in the numerator of the fraction.

Negative Fractions

Recall from Section 2.3 that when we divide two numbers of different signs, the quotient will be negative. For fractions this means that if the numerator is negative and the denominator is positive, the fraction is negative. We could also have the numerator positive and the denominator negative and the fraction will be negative.

That is,

$$\frac{-3}{5} \text{ and } \frac{3}{-5}$$

both represent the negative of the fraction $\frac{3}{5}$.

This means that

$$\frac{-3}{5}, \frac{3}{-5}, \text{ and } -\frac{3}{5}$$

represent the same fraction. Moreover, we generally write negative fractions with the negative sign out in the front of the fraction.

EXAMPLE 5 Simplify the fraction $\frac{-32}{24}$.

SOLUTION: Ignoring the negative sign, the prime factorization of the numerator is

$$32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$$

and the prime factorization of the denominator is

$$24 = 2 \cdot 2 \cdot 2 \cdot 3 = 2^3 \cdot 3.$$

From the prime factorizations note that both the Greatest Common Factor of the numerator and denominator is $2^3 = 8$. That is,

$$\frac{-32}{24} = -\frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 3} = -\frac{8 \cdot 4}{8 \cdot 3}.$$

Eliminating the Greatest Common Factor we get

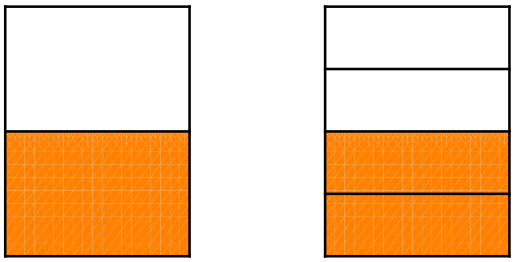
$$\frac{-32}{24} = -\frac{\cancel{8} \cdot 4}{\cancel{8} \cdot 3} = -\frac{4}{3}.$$

Therefore, $\frac{-32}{24}$ simplifies to $-\frac{4}{3}$.

Make the observation that we left the simplified fraction as an improper fraction and did not convert it to a mixed number. In general, we only convert improper fractions to mixed numbers when specifically directed to do so or when working with an application in which an improper fraction does not make sense.

3.3.2 Equivalent Fractions

Consider two rectangles that are the same size with the first divided into two equal parts and the second divided into four equal parts.



The rectangle divided into two equal parts has one part shaded. The shaded area is represented by $\frac{1}{2}$ since one of the two parts of the rectangle is shaded.

The rectangle divided into four equal parts has two parts shaded. The shaded area is represented by $\frac{2}{4}$ since two of the four parts of the rectangle are shaded.

Since both fractions represent the same amount of shaded area, the fractions $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent.

Generally we say that two fractions are **equivalent** if they represent the same quantity. To determine if two or more fractions are equivalent we simply need to write each fraction in simplified form.

EXAMPLE 1 Determine if the fractions $\frac{4}{8}$ and $\frac{6}{12}$ are equivalent.

Avoiding Mistakes

The test to determine whether two fractions are equivalent is not the same process as multiplying fractions. Multiplication of fractions is covered in a future chapter.

SOLUTION: Simplifying both fractions we get

$$\frac{4}{8} = \frac{1 \cdot 4}{2 \cdot 4} = \frac{1 \cdot \cancel{4}}{2 \cdot \cancel{4}} = \frac{1}{2}$$

and

$$\frac{6}{12} = \frac{1 \cdot 6}{2 \cdot 6} = \frac{1 \cdot \cancel{6}}{2 \cdot \cancel{6}} = \frac{1}{2}.$$

Since each fraction simplifies to $\frac{1}{2}$ they are equivalent.

EXAMPLE 2 Determine if the fractions $\frac{3}{36}$ and $\frac{2}{36}$ are equivalent.

SOLUTION: Simplifying both fractions we get

$$\frac{3}{36} = \frac{1 \cdot 3}{3 \cdot 12} = \frac{1 \cdot \cancel{3}}{\cancel{3} \cdot 12} = \frac{1}{12}$$

and

$$\frac{2}{36} = \frac{1 \cdot 2}{2 \cdot 18} = \frac{1 \cdot \cancel{2}}{\cancel{2} \cdot 18} = \frac{1}{18}.$$

Since the fractions do not simplify to the same fraction, they are not equivalent.

3.3.3 Order of Operations and Simplifying Fractions: Rational Expressions

We now turn our attention to fractions in which the numerator and/or the denominator are expressions other than single numbers.

To handle these expressions keep in mind that the numerator and denominator act as a grouping and according to the Order of Operations we need to perform all operations within any groupings first. This means that we need to simplify in the numerator and in the denominator before we can simplify the fraction. Moreover, because we are now working with fractions that have expressions for numerators and/or denominators rather than single numbers, we often refer to them as **Rational Expressions** rather than fractions. We typically use the word fraction to mean a single number divided by a single number.

EXAMPLE 1 Simplify the rational expression $\dfrac{7-2}{17+3}$.

SOLUTION: We need to simplify the numerator and denominator separately first. The rational expression simplifies to

$$7-2=5$$

and the denominator simplifies to

$$17+3=20.$$

We now have that

$$\frac{7-2}{17+3}=\frac{5}{20}$$

so we can now simplify the fraction if possible. Upon inspection, the Greatest Common Factor of the numerator and denominator is 5. So,

$$\frac{5}{20}=\frac{1\cdot 5}{4\cdot 5}=\frac{1\cdot \cancel{5}}{4\cdot \cancel{5}}=\frac{1}{4}.$$

EXAMPLE 2 Simplify the rational expression $\dfrac{32+3}{8-3}$.

SOLUTION: Begin by simplifying the numerator and denominator separately and then simplifying the resulting fraction.

$$\frac{32+3}{8-3}=\frac{35}{5}=7.$$

EXAMPLE 3 Simplify the rational expression $\dfrac{2\cdot 3^2-4\div 2}{15\div 3}$.

SOLUTION: Begin by simplifying the numerator and denominator separately and then simplifying the resulting fraction. Applying the Order of Operations in the numerator we get

$$2\cdot 3^2-4\div 2=2\cdot 9-4\div 2=18-2=16.$$

In the denominator we get

$$15\div 3=5.$$

Therefore,

$$\frac{2\cdot 3^2-4\div 2}{15\div 3}=\frac{16}{5}$$

and because 16 and 5 do not share any factors in common we are done.

Note that you have worked with these expressions already. From the exercise set in Section 2.1 you had #33 and #35, namely

$$50(4+81)\div[10(28-3)] \quad \text{and} \quad [150-(60+5)]\div(10+7).$$

These could be written as rational expressions. That is, they could be written as

$$\frac{50(4+81)}{10(28-3)} \quad \text{and} \quad \frac{150-(60+5)}{10+7}.$$

Section 3.3 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 13, simplify the fraction.

1) $\frac{4}{8}$ 2) $\frac{100}{50}$ 3) $\frac{42}{6}$ 4) $\frac{30}{35}$

5) $\frac{21}{24}$ 6) $\frac{11}{77}$ 7) $\frac{0}{15}$ 8) $\frac{12}{36}$

9) $\frac{2}{52}$ 10) $\frac{16}{64}$ 11) $\frac{500}{100}$ 12) $\frac{3}{123}$

13) $\frac{19}{57}$

For problems 14 - 18, determine if the given fractions are equivalent.

14) $\frac{8}{16}$ and $\frac{1}{2}$ 15) $-\frac{5}{6}$ and $\frac{30}{-36}$ 16) $\frac{6}{42}$ and $\frac{9}{54}$ 17) $\frac{2}{4}$ and $-\frac{10}{20}$

18) $\frac{0}{2}$ and $\frac{0}{3}$

For problems 19 - 25, use the Order of Operations to simplify the expression.

19) $\frac{4-2}{10+14}$ 20) $\frac{12+20}{16-8}$ 21) $\frac{20+12}{8-16}$ 22) $\frac{2+2}{1-1}$

23) $\frac{3^3-(5-12)}{(3-(-3))\cdot(-13-4)}$ 24) $\frac{-16\div 8\cdot 5}{-12}$ 25) $\frac{5-(-2)}{(-30)-5}$

Review. For problems 26 - 29, simplify the expression.

26) 2^3 27) -2^3 28) $(-2)^2$ 29) -2^2

30) $4 \cdot (3+5) - 7$

31) Convert $11\frac{2}{5}$ into an improper fraction.

32) Convert $\frac{57}{5}$ to a mixed number.

33) Find the prime factorization of 15.

34) Find the prime factorization of 150.

35) Find the prime factorization of 360.

Chapter 3
Section 3.3 – Simplifying Fractions and Equivalent Fractions

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

1) _____

2) _____

3) _____

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31) _____

32) _____

33) _____

34) _____

35) _____

INTRODUCTION TO ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF REAL NUMBERS

CHAPTER

4

CHAPTER OBJECTIVES

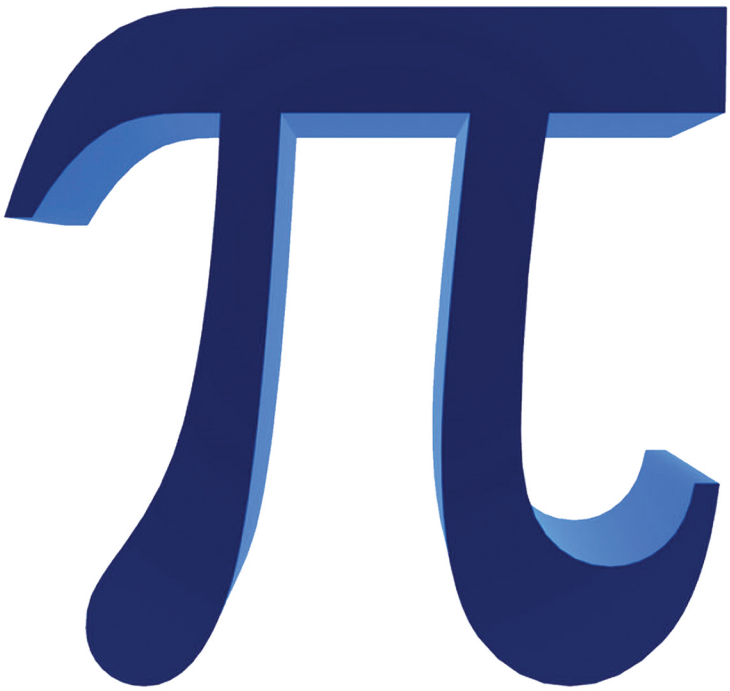
4.1 Addition and Subtraction of Like Fractions

4.2 Least Common Multiple and Least Common Denominator

4.3 Addition and Subtraction of Unlike Fractions

4.4 Multiplication and Division of Fractions

The Pi Symbol



The Pi sign is a mathematical symbol that relates to a transcendental or mystical number which does not really equate to a quantifiable extent or quantity or size. Mathematically expressed, Pi is the ratio of a given circle's circumference to its diameter. Though Pi is roughly equal to 3.1416 for the sake of solving mathematical problems, its real value is a series of numbers that go on infinitely as in 3.14159265358979323846... and on. Mystical Pi symbolized the universe as Pi and the universe produces non-repeating patterns. It has been said that the Pi symbol is a sinister Masonic sign, and only the most elite in the hierarchy of secret masons know what terror lies behind the Pi sign.

4.1 Addition and Subtraction of Like Fractions

In chapters 1 and 2 we discussed how to apply the mathematical operations of addition, subtraction, multiplication, and division with the set of Integers. In chapter 3 we introduced the set of Rational Numbers (fractions). Here, we begin the investigation of how to apply the operations of addition and subtraction with a certain type of Rational Number. In this section we work with fractions that have the same, or common, denominator.

You may recall that any number divided by 1 is itself. That is,

$$3 \div 1 = 3, \text{ or } -7 \div 1 = -7.$$

Moreover, remember that

$$\frac{3}{1} = 3 \div 1 = 3 \text{ and } \frac{-7}{1} = -7 \div 1 = -7.$$

The point being that integers are just fractions with a denominator of 1. One of the reasons we are able to add and subtract integers is that they are all some number of the same "unit." That is, 3 is really "three ones", and -7 is really "the opposite of seven ones" so when we write

$$3 + (-7)$$

we mean "add three ones with the opposite of seven ones." Of course we have been working with integers long enough that we never actually say that but the idea of adding or subtracting numbers with the same units applies in everyday life.

Applications of combining like units.

Suppose you are part of a running club consisting of 6 members and you want to determine the total number of miles your group has run this week.

Four of the members report that they have run a combined 52 miles. The other 2 members report that they have run a combined 45 kilometers.

It would be incorrect to say that as a whole the group has run $52+45=97$ miles. This is wrong because "miles" and "kilometers" are not the same unit and therefore cannot be combined.

Applications of combining like units.

Suppose you are part of a construction crew framing a house and that you are asked to determine how many feet of 2×6's are left at the job site. When you go see what kind of lumber is left, you find that there are piles of 2×4's and 2×6's.

In particular, there are twelve 10-foot 2×4's, and nine 8-foot 2×6's.

The twelve 10-foot 2×4's combine to give 120 feet of 2×4's.

The nine 8-foot 2×6's combine to give 72 feet of 2×6's.

This means that all together there is a total of 192 feet of wood left. However, it would be incorrect to say that there are 192 feet of 2×6's as it would be incorrect to combine the number of feet of 2×4's with the number of feet of 2×6's. They do not represent the same "unit" of wood and so should not be combined.

The idea of combining like units applies to fractions as well.

4.1.1 Like Fractions

In order for fractions to be added or subtracted their denominator must be the same. When two fractions have the same denominator they are called **Like Fractions**.

To connect this idea to numbers with the same units, fractions with the same denominators are considered to be of the same size. That is, just like the integer 3 represents "three ones", the fraction

$$\frac{3}{2}$$

represents "three one-halfs."

Consider the fractions

$$\frac{3}{4} \text{ and } \frac{7}{4}.$$

The fraction $\frac{3}{4}$ represents "three fourths" and the fraction $\frac{7}{4}$ represents "seven fourths." They are considered like fractions because they represent certain numbers of the same unit, namely, the unit "one-fourth."

The two fractions

$$\frac{5}{7} \text{ and } \frac{1}{3}$$

are not like fractions. The fraction $\frac{5}{7}$ represents "five sevenths" while the fraction $\frac{1}{3}$ represents "one third." They do not represent numbers of the same unit, or size.

Like fractions can be added, or subtracted, by following these three steps.

Addition and Subtraction of Like Fractions
To add or subtract fractions with the same denominator,
1) add or subtract the numerators, 2) write the sum, or difference, of the numerators over the common denominator, and 3) simplify the fraction if possible.

The addition and subtraction of fractions that do not have the same, or common, denominator is covered in the next section.

EXAMPLE 1 Perform the indicated operation: $\frac{1}{9}+\frac{7}{9}$.

SOLUTION: Because the two fractions have the same denominator they are like fractions and we can add them. Following the steps for adding like fractions we get

$$\frac{1}{9}+\frac{7}{9}=\frac{1+7}{9}=\frac{8}{9}.$$

Because the numerator and denominator do not share any common factors, the answer is $\frac{8}{9}$.

Avoiding Mistakes

Notice that when adding fractions we do not add the denominators. We only add the numerators.

The interpretation here is that the fraction $\frac{1}{9}$ represents "one ninth" and the fraction $\frac{7}{9}$ represents "seven ninths." If we add them we get

"one ninth" + "seven ninths" = eight ninths.

EXAMPLE 2 Perform the indicated operation: $\frac{10}{7}-\frac{4}{7}$.

SOLUTION: Because the two fractions have the same denominator they are like fractions and we can subtract them. Following the steps for subtracting like fractions we get

$$\frac{10}{7}-\frac{4}{7}=\frac{10-4}{7}=\frac{6}{7}.$$

Because the numerator and denominator do not share any common factors, the answer is $\frac{6}{7}$.

Avoiding Mistakes

Notice that when subtracting fractions we do not subtract the denominators. We only subtract the numerators.

The interpretation here is that the fraction $\frac{10}{7}$ represents "ten sevenths" and the fraction $\frac{4}{7}$ represents "four sevenths." If we subtract them we get

"ten sevenths" − "four sevenths" = "six sevenths".

The addition and subtraction of fractions follow the same rules as addition and subtraction of integers. That is, perform addition and subtraction as they occur from left to right. The next couple of examples illustrate this.

EXAMPLE 3 Perform the indicated operation: $\frac{4}{25}+\frac{17}{25}-\frac{11}{25}$.

SOLUTION: Because the three fractions have the same denominator, they are like fractions and we can add or subtract them. According to the Order of Operations, we perform addition and subtraction as they occur from left to right. Therefore,

$$\frac{4}{25}+\frac{17}{25}-\frac{11}{25}=\frac{4+17}{25}-\frac{11}{25}$$

$$=\frac{21}{25}-\frac{11}{25}$$

$$=\frac{21-11}{25}$$

$$=\frac{10}{25}.$$

Because the numerator and denominator share factors in common we need to simplify the result.

$$\frac{10}{25}=\frac{5\cdot 2}{5\cdot 5}=\frac{\cancel{5}\cdot 2}{\cancel{5}\cdot 5}=\frac{2}{5}$$

Therefore,

$$\frac{4}{25}+\frac{17}{25}-\frac{11}{25}=\frac{4+17}{25}-\frac{11}{25}=\frac{2}{5}.$$

An important note here is that while the fractions $\frac{10}{25}$ and $\frac{2}{5}$ are equivalent fractions it would have been incorrect to say that $\frac{10}{25}$ is the answer as it is not simplified.

★ Always simplify fractions when possible.

EXAMPLE 4 Perform the indicated operation: $-\frac{5}{6}+\frac{1}{6}+\frac{7}{6}-\frac{1}{6}$.

SOLUTION: Every fraction has the same denominator so they are all like fractions and we are able to add or subtract them. Working from left to right we get

$$\frac{-5}{6}+\frac{1}{6}+\frac{7}{6}-\frac{1}{6}=\frac{-5+1}{6}+\frac{7}{6}-\frac{1}{6}$$

$$=\frac{-4}{6}+\frac{7}{6}-\frac{1}{6}$$

$$=\frac{-4+7}{6}-\frac{1}{6}$$

$$=\frac{3}{6}-\frac{1}{6}$$

$$=\frac{3-1}{6}$$

$$=\frac{2}{6}$$

$$=\frac{2\cdot 1}{2\cdot 3}$$

$$=\frac{\cancel{2}\cdot 1}{\cancel{2}\cdot 3}$$

$$=\frac{1}{3}.$$

The next example illustrates an alternative approach to problems like these that may be more efficient.

EXAMPLE 5 Perform the indicated operation: $-\frac{7}{12}+\frac{5}{12}-\frac{1}{12}+\frac{17}{12}$.

SOLUTION: Since the fractions are all like fractions,

$$-\frac{7}{12}+\frac{5}{12}-\frac{1}{12}+\frac{17}{12}$$

$$=\frac{-7+5-1+17}{12}$$

$$=\frac{14}{12}$$

$$=\frac{2\cdot 7}{2\cdot 6}$$

$$=\frac{\cancel{2}\cdot 7}{\cancel{2}\cdot 6}$$

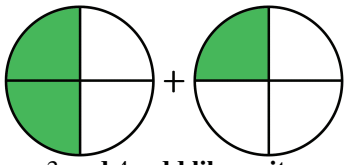
$$=\frac{7}{6}.$$

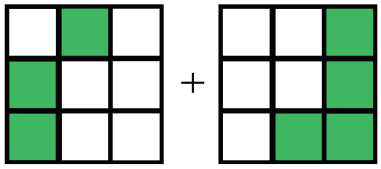
★ Note again that it would have been incorrect to say the answer to example 5 was $\frac{14}{12}$ since it is not simplified.

Section 4.1 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For exercises 1 and 2, shade in the portion of the figure that represents the addition of the first two figures.

1) 

2) 

For problems 3 **and** 4**, add like units.**

3) 4 cats + 3 cats

4) 6 computers + 7 computers

For problems 5 – 10**, add like fractions. Express your answer as a fraction or a whole number.**

5) $\frac{3}{4}+\frac{7}{4}$

6) $\frac{2}{5}+\frac{8}{5}$

7) $\frac{1}{6}+\frac{2}{6}+\frac{5}{6}$

8) $\frac{4}{9}+\frac{5}{9}+\frac{7}{9}$

9) $\frac{2}{7}+\frac{4}{7}+\frac{8}{7}$

10) $\frac{5}{24}+\frac{12}{24}+\frac{1}{24}$

11) Aubrey walked $\frac{1}{6}$ of a mile on the way to work and then ran the remaining $\frac{5}{6}$ of a mile to arrive at work on time. How far did Aubrey have to travel to get to work?

12) There are 27 students in Dr. Sharpe's math class. Four students earned an A, seven students earned a B, and six students earned a C. What fraction of Dr. Sharpe's students passed?

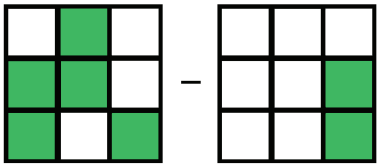
13) There are 31 students in Professor Lange's class. Four students withdrew, and five students failed. What fraction of students passed Professor Lange's class?

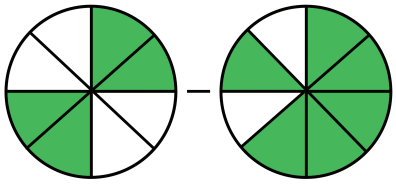
For problems 14 and 15, subtract like units.

14) 65 motorcycles - 23 motorcycles

15) 195 chicken wings - 99 chicken wings

For problems 16 **and** 17**, shade in the portion of the figure that represents the subtraction of the first two figures.**

16) 

17) 

For problems 18 - 23, subtract like fractions. Express your answer as a fraction or a whole number.

18) $\frac{6}{7}-\frac{3}{7}$

19) $\frac{5}{8}-\frac{3}{8}$

20) $\frac{29}{30}-\frac{2}{30}-\frac{15}{30}$

21) $\frac{34}{37}-\frac{30}{37}-\frac{1}{37}$

22) $\frac{33}{40}-\frac{23}{40}-\frac{7}{40}$

23) $\frac{55}{56}-\frac{45}{56}-\frac{9}{56}$

For problems 24 - 28, add or subtract as indicated. Express your answer as a fraction or a whole number.

24) $\frac{3}{23}+\frac{16}{23}$

25) $\frac{4}{27}+\frac{11}{27}+\frac{12}{27}$

26) $\frac{45}{56}-\frac{23}{56}+\frac{5}{56}$

27) $\frac{78}{99}+\frac{10}{99}-\frac{75}{99}$

28) $\frac{24}{7}-\frac{17}{7}-\frac{1}{7}$

For problems 29 - 31, use the Order of Operations to simplify.

29) $\frac{5+3^2-4^2}{|-16|\div 2}$

30) $2\left[-(2+3)^2\right]\div(9+4)$

31) $-\frac{(-2)^3+2^3}{5(6-11)}$

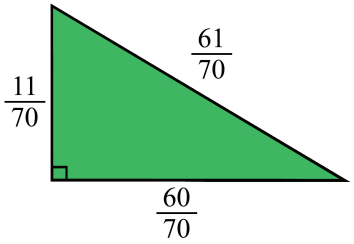
For problems 32 - 34, write the mixed number as an improper fraction.

32) $5\frac{2}{7}$

33) $1\frac{7}{9}$

34) $8\frac{3}{5}$

35) Find the perimeter of the figure.

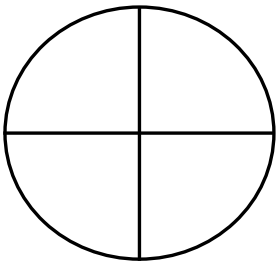


Chapter 4
Section 4.1 – Addition and Subtraction of Like Fractions

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

1)



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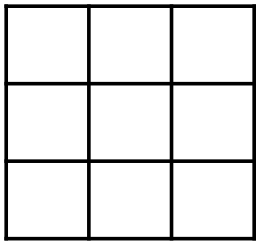

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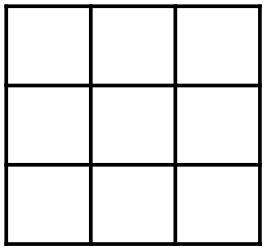

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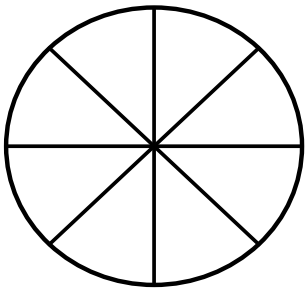

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33) _____

34) _____

35) _____

4.2 Least Common Multiple and Least Common Denominator

In Section 4.1 we discussed adding and subtracting fractions that have the same denominator. In this section we look at developing the skills that will allow us to add and subtract fractions with different denominators, unlike fractions. When asked to add or subtract unlike fractions, we will need to write each fraction with a common denominator. In other words, we will write unlike fractions as like fractions. To accomplish this we need to discuss the Least Common Multiple of two numbers.

4.2.1 Least Common Multiple

The **multiples** of an integer are found by multiplying the integer by natural numbers. For example, the multiples of 4 are

$$4 \cdot 1 = 4,\ 4 \cdot 2 = 8,\ 4 \cdot 3 = 12,\ 4 \cdot 4 = 16, \ldots.$$

EXAMPLE 1 Find the first five multiples of the number 8.

SOLUTION: To find the multiples of 8 we multiply it by 1, 2, 3, 4, ... The first five multiples of 8 are then

$$8(1) = 8,\ 8(2) = 16,\ 8(3) = 24,\ 8(4) = 32,\ \text{and}\ 8(5) = 40.$$

EXAMPLE 2 Find the first five multiples of the number 3.

SOLUTION: To find the multiples of 3 we multiply it by 1, 2, 3, 4, ... The first five multiples of 3 are then

$$3(1) = 3,\ 3(2) = 6,\ 3(3) = 9,\ 3(4) = 12,\ \text{and}\ 3(5) = 15,$$

The **Least Common Multiple** (LCM) of two numbers is the smallest number that is a multiple of both numbers. There are two methods we can use to find the LCM of two numbers.

The first method creates a list of multiples for each number. The LCM will be the smallest multiple they share in common.

The second method relies on finding the prime factorization of the two numbers.

Finding the LCM with Lists of Multiples

EXAMPLE 1 Find the LCM of 6 and 10.

SOLUTION: Make a list of the multiples of each number. The multiples of 6 are

$$6, 12, 18, 24, 30, , 36, \ldots$$

The multiples of 10 are

$$10, 20, 30, 40, \ldots$$

Make the observation that the smallest number that is a multiple of both 6 and 10 is 30. So the LCM of 6 and 10 is 30.

EXAMPLE 2 Find the LCM of 12 and 15.

SOLUTION: Make a list of the multiples of each number. The multiples of 12 are

$$12, 24, 36, 48, 60, 72, \ldots$$

The multiples of 15 are

$$15, 30, 45, 60, \ldots$$

Make the observation that the smallest number that is a multiple of both 12 and 15 is 60. So the LCM of 12 and 15 is 60.

Finding the LCM using Prime Factorizations

This method is best explained with examples and then generalized.

EXAMPLE 1 Find the LCM of 16 and 24.

SOLUTION: Write the prime factorization of both numbers.

$$16 = 2^4, \text{ and } 24 = 2^3 \cdot 3 = 2^3 \cdot 3^1.$$

To find the LCM, we consider each factor in the prime factorizations.

Each prime factorization contains the factor 2. Because the LCM of both numbers has to contain both numbers, the LCM will contain the factor 2 raised to the highest power that occurs in each of the prime factorizations.

The prime factorization of 16 contains 2^4 and the prime factorization of 24 contains 2^3 so the highest power on 2 in each prime factorization is 4. This means that the LCM will contain 2^4.

The next factor in the prime factorizations is 3. Again, because the LCM of both numbers has to contain both numbers, the LCM will contain the factor 3 raised to the highest power that occurs in each of the prime factorizations.

The prime factorization of 16 does not contain the factor 3, but the prime factorization of 24 contains 3^1. Therefore, the highest power on 3 in each prime factorization is 1, so the LCM will contain 3^1.

Putting these together, we see that the LCM of 16 and 24 is

$$2^4 \cdot 3^1 = 48.$$

With example 1 in mind, the LCM of two numbers will be the product of each factor contained in the prime factorizations of the numbers raised to the highest power that occurs in each of the prime factorizations.

EXAMPLE 2 Find the LCM of 90 and 75.

SOLUTION: The prime factorization of both numbers is

$$90 = 2 \cdot 3^2 \cdot 5, \text{ and } 75 = 3 \cdot 5^2.$$

The highest power on the factor 2 is 1 so the LCM will contain 2^1.

The highest power on the factor 3 is 2 so the LCM will contain 3^2.

The highest power on the fact 5 is 2 so the LCM will contain 5^2.

Therefore, the LCM of 90 and 75 is $2 \cdot 3^2 \cdot 5^2 = 450$.

An alternative method for finding the LCM of two or more numbers will be used in section 4.2.3.

4.2.2 Writing Fractions with an Indicated Denominator

In section 3.3 we studied how to simplify fractions. As an example, the fraction $\frac{15}{10}$ simplifies to

$$\frac{15}{10} = \frac{3 \cdot 5}{2 \cdot 5} = \frac{3 \cdot \cancel{5}}{2 \cdot \cancel{5}} = \frac{3}{2}.$$

Here we look at going the other direction. That is, we now ask you to write the fraction $\frac{3}{2}$ with a denominator of 10.

In general, to write a fraction with some indicated denominator, divide the new denominator by the original denominator and multiply the numerator and denominator by the result.

EXAMPLE 1 Write the fraction $\frac{4}{15}$ with a denominator of 75.

SOLUTION: Dividing the new denominator by the original denominator results in

$$\frac{75}{15} = 5.$$

To write the fraction $\frac{4}{15}$ with a denominator of 75, multiply the numerator and denominator by 5. That is,

$$\frac{4}{15} = \frac{4 \cdot 5}{15 \cdot 5} = \frac{20}{75}.$$

Note again that in section 3.3 we just went the other way. That is, in section 3.3 we simplified.

$$\frac{20}{75} = \frac{4 \cdot 5}{15 \cdot 5} = \frac{4 \cdot \cancel{5}}{15 \cdot \cancel{5}} = \frac{4}{15}$$

Here we are using the same technique for a different purpose.

EXAMPLE 2 Write the fraction with the indicated denominator: $\frac{11}{17} = \frac{\quad}{51}$.

SOLUTION: Here we are being asked to write

$$\frac{11}{17}$$

with a denominator of 51. Since $51 \div 17 = 3$ we have

$$\frac{11}{17} = \frac{11 \cdot 3}{17 \cdot 3} = \frac{33}{51}.$$

4.2.3 Least Common Denominator

The **Least Common Denominator** (LCD) of two fractions is the Least Common Multiple (LCM) of their denominators.

EXAMPLE 1 Find the LCD of the fractions $\frac{5}{6}$ and $\frac{1}{9}$ and write each fraction with the LCD as the denominator.

SOLUTION: The LCD of the fractions will be the LCM of their denominators. The prime factorizations of the denominators are

$$6 = 2^1 \cdot 3^1 \text{ and } 9 = 3^2.$$

The highest power on the factor 2 is 1 so the LCM will contain 2^1 and the highest power on the factor 3 is 2 so the LCM will contain 3^2. So, the LCD is

$$2^1 \cdot 3^2 = 18.$$

To write $\frac{5}{6}$ with a denominator of 18 we can either look at the prime factorization of the LCD and see that the denominator is missing a factor of 3, or note that $18 \div 6 = 3$. Either way,

$$\frac{5}{6} = \frac{5 \cdot 3}{6 \cdot 3} = \frac{15}{18}$$

To write $\frac{1}{9}$ with a denominator of 18 we can either look at the prime factorization of the LCD and see that the denominator is missing a factor of 2, or note that $18 \div 9 = 2$. Either way,

$$\frac{1}{9} = \frac{1 \cdot 2}{9 \cdot 2} = \frac{2}{18}$$

The LCD of $\frac{5}{6}$ and $\frac{1}{9}$ is 18, and

$$\frac{5}{6} = \frac{15}{18} \text{ and } \frac{1}{9} = \frac{2}{18}.$$

The next example illustrates an alternative method that can be used to find the LCM, and by extension the LCD, of two numbers. The method uses a table of multiples.

EXAMPLE 2 Find the LCD of the fractions $\frac{7}{8}$, and $\frac{1}{6}$. Write each fraction with the LCD as the denominator.

SOLUTION: From the table of multiples,

Multiples of 8	Multiples of 6
8	6
16	12
(24)	18
	(24)

we see that the LCM of the denominators is 24 and so the LCD of the fractions is 24.

To write the fraction $\frac{7}{8}$ with a denominator of 24 note that $24 \div 8 = 3$ so

$$\frac{7}{8} = \frac{7 \cdot 3}{8 \cdot 3} = \frac{21}{24}.$$

For $\frac{1}{6}$, $24 \div 6 = 4$ so

$$\frac{1}{6} = \frac{1 \cdot 4}{6 \cdot 4} = \frac{4}{24}.$$

EXAMPLE 3 Find the LCD of the fractions $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{7}{15}$. Write each fraction with the LCD as the denominator.

SOLUTION: From the table of multiples,

Multiples of 2	Multiples of 3	Multiples of 15
2	3	15
4	6	(30)
⋮	⋮	
18	(30)	
⋮		
(30)		

we see that the LCM of the denominators is 30 and so the LCD of the fractions is 30.

To write the fraction $\frac{1}{2}$ with a denominator of 30 note that $30 \div 2 = 15$ so

$$\frac{1}{2} = \frac{1 \cdot 15}{2 \cdot 15} = \frac{15}{30}.$$

For $\frac{2}{3}$, $30 \div 3 = 10$ so

$$\frac{2}{3} = \frac{2 \cdot 10}{3 \cdot 10} = \frac{20}{30}.$$

For $\frac{7}{15}$, $30 \div 15 = 2$ so

$$\frac{7}{15} = \frac{7 \cdot 2}{15 \cdot 2} = \frac{14}{30}.$$

4.2.4 Ranking Fractions From Least to Greatest

In order to rank fractions, they must all have the same denominator. Once all of the fractions have the same denominator, they can be ranked, or ordered, according to their numerators.

EXAMPLE 1 Rank the fractions $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{7}{15}$ from least to greatest.

SOLUTION: From the previous example we know that

$$\frac{1}{2} = \frac{15}{30}, \frac{2}{3} = \frac{20}{30}, \text{ and } \frac{7}{15} = \frac{14}{30}.$$

To rank the fractions $\frac{15}{30}$, $\frac{20}{30}$, and $\frac{14}{30}$ from least to greatest we need to rank the numerators from least to greatest. That is since,

$$14 \leq 15 \leq 20,$$

it must be that

$$\frac{14}{30} \leq \frac{15}{30} \leq \frac{20}{30}.$$

So, from least to greatest, we get $\frac{7}{15}$, $\frac{1}{2}$, and $\frac{2}{3}$.

EXAMPLE 2 Rank the fractions $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{7}$ from least to greatest.

SOLUTION: To find the LCD note the table of multiples for 3, 4, and 7.

Multiples of 3	Multiples of 4	Multiples of 7
3	4	7
6	8	14
9	12	21
⋮	⋮	⋮
81	80	77
(84)	(84)	(84)

From the table we see that the LCD is 84. Also note that because none of the denominators share any common factors the LCD would have to be

$$3(4)(7) = 84.$$

Writing each fraction with a denominator of 84 gives

$$\frac{2}{3} = \frac{2 \cdot 28}{3 \cdot 28} = \frac{56}{84},$$

$$\frac{3}{4} = \frac{3 \cdot 21}{4 \cdot 21} = \frac{63}{84},$$

$$\text{and } \frac{4}{7} = \frac{4 \cdot 12}{7 \cdot 12} = \frac{48}{84}.$$

Then, based on the numerators,

$$\frac{48}{84} \leq \frac{56}{84} \leq \frac{63}{84}.$$

So, from least to greatest we have $\frac{4}{7}$, $\frac{2}{3}$, and $\frac{3}{4}$.

Section 4.2 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 4, list the first six multiples of the given number.

1) 4 2) 7 3) 12 4) 30

For problems 5 - 11, find the LCM of the given numbers.

5) 15 and 20 6) 3 and 6 7) 32 and 36 8) 8, 12, and 20

9) 3, 15, and 21 10) 14, 21, and 30 11) 8, 14, 21, and 32

For problems 12 - 14, write each fraction with the indicated denominator.

12) $\frac{5}{6} = \frac{}{42}$ 13) $\frac{1}{7} = \frac{}{35}$ 14) $\frac{7}{8} = \frac{}{32}$

For problems 15 - 18, rank the fractions from least to greatest.

15) $\frac{5}{16}, \frac{7}{24}, \frac{9}{28}$ 16) $\frac{9}{7}, \frac{11}{9}, \frac{19}{18}$ 17) $\frac{3}{5}, \frac{4}{7}, \frac{59}{70}$ 18) $\frac{6}{7}, \frac{7}{10}, \frac{13}{14}, \frac{17}{20}$

19) Five satellites revolve around the earth once every 4, 6, 9, 12, and 15 hours, respectively. How many hours must pass before the satellites are lined up?

20) At a party, Marc had $\frac{2}{5}$ lb of broccoli , $\frac{1}{4}$ lb of baby spinach, and $\frac{1}{2}$ lb of celery. Which type of vegetable did he eat the most of? Which type of vegetable did he eat the least of?

21) Which of the following fractions is between $\frac{1}{8}$ and $\frac{5}{12}$?

a) $\frac{1}{6}$ b) $\frac{19}{36}$ c) $\frac{7}{14}$ d) $\frac{5}{24}$

22) Which of the following fractions is between $\frac{3}{5}$ and $\frac{7}{5}$?

a) $\frac{71}{100}$ b) $\frac{69}{50}$ c) $\frac{29}{50}$ d) $\frac{16}{25}$

Review. Simplify the expressions using the Order of Operations.

23) $\frac{5}{6}-\frac{1}{6}$ 24) $\frac{7}{3}+\frac{2}{3}$ 25) $\frac{1}{5}+\frac{4}{5}-\frac{3}{5}$

Review. Simplify the given fraction or rational expression.

26) $\frac{35}{15}$ 27) $-\frac{24}{36}$ 28) $\frac{168}{210}$ 29) $\frac{92}{184}$

30) $\frac{3+5}{37-5}$ 31) $\frac{|5-13|+6}{28}$ 32) $\frac{-5}{2^4-1}$ 33) $\frac{60}{210}$

Translate the English expression into a mathematical expression and simplify.

34) The product of negative seven and thirteen plus three.

35) The sum of eight and six divided by four.

Chapter 4
Section 4.2 – Least Common Multiple and Least Common Denominator

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

1) _____

2) _____

3) _____

4) _____

5) _____

6) _____

7) _____

8) _____

9) _____

10) _____

11) _____

12) _____

13) _____

14) _____

15) _____

16) _____

17) _____

18) _____

19) _____

20) _____

21) _____

22) _____

23) _____

24) _____

25) _____

26) _____

27) _____

28) _____

29) _____

30) _____

31) _____

32) _____

33) _____

34) _____

35) _____

4.3 Addition and Subtraction of Unlike Fractions

Unlike fractions are fractions that have different denominators. In order to add or subtract unlike fractions, we need to write the fractions so they have a common denominator.

While any common denominator (any common multiple of the denominators) will work, we will always use the Least Common Denominator (Least Common Multiple of the denominators).

★ The important thing to remember is that in order to add or subtract fractions they must have the same denominator.

EXAMPLE 1 Perform the indicated operation(s): $\frac{1}{3}+\frac{1}{4}$.

SOLUTION: The Least Common Multiple of the denominators 3 and 4 is 12 so the Least Common Denominator, LCD, is 12. The first step in performing the addition is to write each fraction with a denominator of 12. Using the technique from the previous section we get that

$$\frac{1}{3}=\frac{1\cdot 4}{3\cdot 4}=\frac{4}{12} \quad \text{and} \quad \frac{1}{4}=\frac{1\cdot 3}{4\cdot 3}=\frac{3}{12}.$$

Now that both fractions have the same denominator we follow the rule for adding and subtracting like fractions. That is,

$$\frac{1}{3}+\frac{1}{4}=\frac{4}{12}+\frac{3}{12}$$

$$=\frac{4+3}{12}$$

$$=\frac{7}{12}.$$

The last step would be to simplify the fraction if possible. Since 7 and 12 do not share any factors in common, the answer is $\frac{7}{12}$.

EXAMPLE 2 Perform the indicated operation(s): $\frac{5}{12}-\frac{4}{15}$.

SOLUTION: The LCM of the denominators 12 and 15 is 60 so the LCD is 60. Write each fraction with a denominator of 60.

$$\frac{5}{12}=\frac{5\cdot 5}{12\cdot 5}=\frac{25}{60} \text{ and } \frac{4}{15}=\frac{4\cdot 4}{15\cdot 4}=\frac{16}{60}$$

Avoiding Mistakes

In the problem

$$\frac{5}{12}-\frac{4}{15},$$

do not try to "cancel" the 5 and 15. The fractions are being subtracted, not multiplied.

Now that both fractions have the same denominator, we follow the rule for adding and subtracting like fractions. That is,

$$\frac{5}{12}-\frac{4}{15}=\frac{25}{60}-\frac{16}{60}$$

$$=\frac{25-16}{60}$$

$$=\frac{9}{60}$$

Simplifying this result we get

$$\frac{9}{60}=\frac{3\cdot 3}{3\cdot 20}$$

$$=\frac{\cancel{3}\cdot 3}{\cancel{3}\cdot 20}$$

$$=\frac{3}{20}.$$

Therefore, $\frac{5}{12}-\frac{4}{15}=\frac{3}{20}$.

The addition and subtraction of more than two fractions follows the same Order of Operations as integers. This means that we still perform addition and subtraction as they occur from left to right.

EXAMPLE 3 Perform the indicated operation(s): $\frac{3}{5}-\frac{3}{8}+\frac{1}{10}$.

SOLUTION: The LCM of the denominators 5, 8, and 10 is 40. Write each fraction with a denominator of 40 and then perform addition and subtraction of like fractions as they occur from left to right.

$$\frac{3}{5}-\frac{3}{8}+\frac{1}{10}=\frac{3\cdot 8}{5\cdot 8}-\frac{3\cdot 5}{8\cdot 5}+\frac{1\cdot 4}{10\cdot 4}$$

$$=\frac{24}{40}-\frac{15}{40}+\frac{4}{40}$$

$$=\frac{24-15}{40}+\frac{4}{40}$$

$$=\frac{9}{40}+\frac{4}{40}$$

$$=\frac{9+4}{40}$$

$$=\frac{13}{40}$$

Since $\frac{13}{40}$ is simplified, we are done.

Notice that once all of the fractions are written with a common denominator we can perform the addition and subtraction from left to right in the numerator. That is,

$$\frac{3}{5}-\frac{3}{8}+\frac{1}{10}=\frac{24}{40}-\frac{15}{40}+\frac{4}{40}=\frac{24-15+4}{40}$$

$$=\frac{13}{40}.$$

EXAMPLE 4 Perform the indicated operation(s): $-\frac{5}{9}-2+\frac{1}{6}$.

SOLUTION: Remember that Integers are just Rational Numbers with a denominator of 1.

This means that

$$2=\frac{2}{1}$$

and since the LCM of the denominators 9, 1, and 6 is 18, the LCD is 18.

Therefore,

$$-\frac{5}{9}-2+\frac{1}{6}$$

$$=\frac{-5}{9}-\frac{2}{1}+\frac{1}{6}$$

$$=\frac{-5\cdot 2}{9\cdot 2}-\frac{2\cdot 18}{1\cdot 18}+\frac{1\cdot 3}{6\cdot 3}$$

$$=\frac{-10}{18}-\frac{36}{18}+\frac{3}{18}$$

$$=\frac{-10-36+3}{18}$$

$$=\frac{-43}{18}.$$

The answer is $-\frac{43}{18}$.

Notice that we did not convert the improper fraction to a mixed number. The next subsection illustrates how to handle the addition and subtraction of mixed numbers. In general, if the original problem does not have mixed numbers, we do not write the answer as a mixed number.

4.3.1 Mixed Numbers

Recall from section 3.1 that we can convert the the mixed number $a\frac{b}{c}$ to an improper fraction by calculating

$$\frac{ac+b}{c}.$$

The reason behind why this works is that the mixed number $a\frac{b}{c}$ really means

$$a+\frac{b}{c}.$$

We now know that this can be written as

$$\frac{a}{1}+\frac{b}{c}$$

and with an LCD of c this becomes

$$\frac{a\cdot c}{c}+\frac{b}{c}=\frac{a(c)+b}{c}.$$

In particular,

$$3\frac{5}{9}=3+\frac{5}{9}$$

$$=\frac{3}{1}+\frac{5}{9}$$

$$=\frac{3\cdot 9}{9}+\frac{5}{9}$$

$$=\frac{27+5}{9}$$

$$=\frac{32}{9}.$$

By switching mixed numbers to improper fractions, we can add and subtract mixed numbers just as we add and subtract fractions.

EXAMPLE 1 Perform the indicated operation(s): $4\frac{2}{3}-1\frac{3}{5}$. Write the answer as a mixed number if applicable.

SOLUTION: Write each mixed number as an improper fraction.

$$4\frac{2}{3}=\frac{4}{1}+\frac{2}{3}=\frac{4(3)+2}{3}=\frac{14}{3}$$

and

$$1\frac{3}{5}=\frac{1}{1}+\frac{3}{5}=\frac{1(5)+3}{5}=\frac{8}{5}.$$

Now we have that $4\frac{2}{3}-1\frac{3}{5}$ can be written as

$$\frac{14}{3}-\frac{8}{5}.$$

Since the LCD is 15 we get

$$\frac{14}{3}-\frac{8}{5}$$

$$=\frac{14\cdot 5}{3\cdot 5}-\frac{8\cdot 3}{5\cdot 3}$$

$$=\frac{70}{15}-\frac{24}{15}$$

$$=\frac{70-24}{15}$$

$$=\frac{46}{15}.$$

Converting to a mixed number we get $\frac{46}{15}=3\frac{1}{15}$.

EXAMPLE 2 Perform the indicated operation(s): $2\frac{4}{9}+3\frac{1}{6}-5\frac{2}{3}$. Write the answer as a mixed number if applicable.

SOLUTION: Convert the mixed numbers to improper fractions.

$$2\frac{4}{9}+3\frac{1}{6}-5\frac{2}{3}=\frac{22}{9}+\frac{19}{6}-\frac{17}{3}$$

Write the fractions with a common denominator (the LCD is 18).

$$\frac{22}{9}+\frac{19}{6}-\frac{17}{3}=\frac{44}{18}+\frac{57}{18}-\frac{102}{18}$$

Perform addition and subtraction as they occur from left to right.

$$\frac{44}{18}+\frac{57}{18}-\frac{102}{18}=\frac{44+57-102}{18}$$

$$=-\frac{1}{18}$$

Therefore, $2\frac{4}{9}+3\frac{1}{6}-5\frac{2}{3}=-\frac{1}{18}$.

4.3.2 Applications

EXAMPLE 1 David spends $\frac{2}{5}$ of his monthly income on daycare and $\frac{1}{4}$ on food. What fraction of his monthly income does David have left after paying for daycare and food?

SOLUTION: To answer this question we need to first determine the total fraction of his income spent on daycare and food. We find this by adding $\frac{2}{5}$ and $\frac{1}{4}$.

That is, Dave spends a combined

$$\frac{2}{5}+\frac{1}{4}=\frac{8}{20}+\frac{5}{20}=\frac{13}{20}$$

of his monthly income on daycare and food. To determine what fraction he has left, we subtract this from one. Therefore, David has

$$1-\frac{13}{20}=\frac{20}{20}-\frac{13}{20}=\frac{7}{20}$$

of his monthly income left after paying for daycare and food.

EXAMPLE 2 Karen has a triangular portion of her yard she wishes to fence in. She measures each side and finds they measure $12\frac{1}{3}$ feet, $7\frac{3}{4}$ feet, and $9\frac{1}{8}$ feet. How much feet of fencing will she need?

SOLUTION: To determine the total feet of fencing we need to add the lengths. That is, Karen will need a total of

$$12\frac{1}{3}+7\frac{3}{4}+9\frac{1}{8}=\frac{37}{3}+\frac{31}{4}+\frac{73}{8}$$

$$=\frac{296}{24}+\frac{186}{24}+\frac{219}{24}$$

$$=\frac{701}{24}$$

$$=29\frac{5}{24}$$

Therefore we see that she will need $29\frac{5}{24}$ feet of fencing.

Avoiding Mistakes

It is always a good idea to check your answer in the context of the problem to determine if your answer is reasonable.

Section 4.3 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 11, use the Order of Operations to simplify the expression.

1) $\frac{3}{22}+\frac{1}{11}$ 2) $\frac{9}{8}-\frac{7}{9}$ 3) $4+\frac{13}{12}$ 4) $\frac{1}{3}+\frac{5}{9}+\frac{7}{27}$

5) $3-\frac{1}{6}-\frac{5}{12}$ 6) $\frac{1}{4}+\frac{5}{6}-1$ 7) $\frac{3}{8}-\frac{5}{6}+\frac{7}{12}$ 8) $\frac{12}{5}+\frac{7}{10}-\frac{9}{20}-\frac{1}{40}$

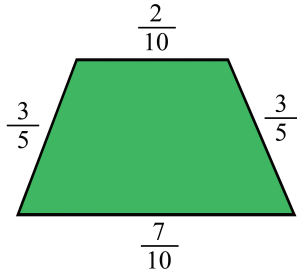
9) $3\frac{1}{3}-2\frac{3}{4}$ 10) $-5\frac{6}{7}+4\frac{3}{7}$ 11) $1\frac{2}{9}-5\frac{1}{3}$

12) Tom has an old car that is leaking several fluids. On Wednesday he added $\frac{1}{2}$ quart oil, $\frac{3}{4}$ quarts power steering fluid, $\frac{1}{8}$ quart of transmission fluid, and $\frac{7}{8}$ quarts of windshield washer fluid. How much total fluid did Tom add to his car?

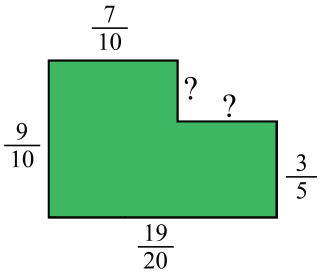
13) Bill hired two men to fix his roof. One fixed $\frac{3}{8}$ of the roof and the other fixed $\frac{1}{4}$ of the roof. Did the roof get fixed? If not, how much of the roof still needs to be fixed?

14) Professor Sharpe gave a mathematics test on Friday. One-third of the students made an A on the test, $\frac{1}{6}$ of the students made a B, and $\frac{1}{6}$ of the students made a C. What fraction of the students did not pass the test?

15) Find the perimeter of the figure.



16) Find the missing dimensions and then find the perimeter.



Review.

17) Find the LCM of 12, 20, and 30.

18) Find the LCM of 15, 27, and 35.

19) Rank the fractions from least to greatest: $\frac{5}{8}, \frac{4}{3}, \frac{7}{6}$.

20) Rank the fractions from least to greatest: $\frac{7}{4}, \frac{6}{5}, \frac{11}{6}, \frac{13}{10}$.

21) Simplify: $\frac{4}{15} + \frac{17}{15} - \frac{6}{15}$

22) Simplify: $\frac{13}{30} + \frac{19}{30} + \frac{28}{30}$

23) Simplify: $\frac{13}{14} - \frac{7}{14} - \frac{11}{14}$

24) Simplify: $6\frac{1}{2} - 3 + 7\frac{2}{3} - 8\frac{3}{4} + 1$

Chapter 4
Section 4.3 – Addition and Subtraction of Unlike Fractions

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

1) _____

2) _____

3) _____

4) _____

5) _____

6) _____

7) _____

8) _____

9) _____

10) _____

11) _____

12) _____

13) _____

14) _____

15) _____

16) Missing: _____ Perimeter: _____

17) _____

18) _____

19) _____

20) _____

21) _____

22) _____

23) _____

24) _____

4.4 Multiplication and Division of Fractions

4.4.1 Multiplication of Fractions

Multiplying a Fraction and an Integer

We know that "one half of 10" is 5 and we know that "one third of 12" is 4. We can write these statements using fractions. That is,

$$\text{"one half of 10"} = \frac{1}{2} \times 10 = 5$$

and

$$\text{"one third of 12"} = \frac{1}{3} \times 12 = 4.$$

We also know that in order to calculate "one half of 10" and "one third of 12" we divide 10 by 2 and 12 by 3. This means that

$$\frac{1}{2} \times 10 = \frac{10}{2} = 5$$

and

$$\frac{1}{3} \times 12 = \frac{12}{3} = 4.$$

We can also look at this the way we did in chapters 1 and 2. That is, since multiplication means repeated addition, $\frac{1}{2} \times 10$ means add $\frac{1}{2}$ to itself 10 times. So,

$$\frac{1}{2} \times 10 = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

$$= \frac{1+1+1+1+1+1+1+1+1+1}{2}$$

$$= \frac{10}{2} = 5.$$

Likewise,

$$\frac{1}{3}\times 12 = \frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$$

$$= \frac{1+1+1+1+1+1+1+1+1+1+1+1}{3}$$

$$= \frac{12}{3} = 4.$$

This can be extended to expressions like $\frac{2}{3}\times 4$ and $\frac{3}{5}\times 2$. That is,

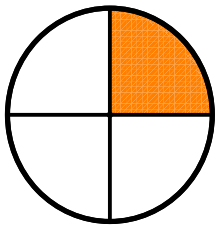
$$\frac{2}{3}\times 4 = \frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3} = \frac{2+2+2+2}{3} = \frac{2(4)}{3} = \frac{8}{3}$$

and

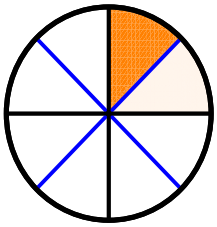
$$\frac{3}{5}\times 2 = \frac{3}{5}+\frac{3}{5} = \frac{3+3}{5} = \frac{3(2)}{5} = \frac{6}{5}.$$

Multiplying two Fractions

We can visualize what's going on when we multiply two fractions. In the figure shown here, one fourth, or $\frac{1}{4}$, of the circle is shaded.

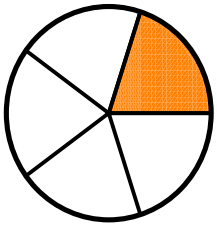


If we wanted to know what the fractional value of one half of one quarter of the original circle is, we could cut the shaded quarter, and all other quarters, in half and count how many parts of the circle are shaded.

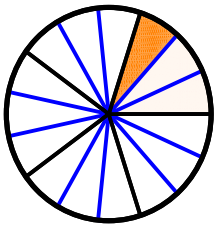


When we do this we see that one out of the eight slices is shaded. This means that $\frac{1}{2}$ of $\frac{1}{4}$ is $\frac{1}{8}$.

What if we wanted to calculate $\frac{2}{3}$ of $\frac{1}{5}$? Since $\frac{2}{3}$ is really "two thirds", let's first look at what one third of $\frac{1}{5}$ is. The figure here shows that $\frac{1}{5}$ of a circle is shaded.



Because we want "one third" of this $\frac{1}{5}$, divide the fifths into three equal parts and only take one third of the shaded one fifth slice.



If we count up the number of slices now we see that one of 15 slices is shaded. This means that $\frac{1}{3}$ of $\frac{1}{5}$ is $\frac{1}{15}$.
Because we wanted two thirds of $\frac{1}{5}$, we need two $\frac{1}{15}$'s or

$$\frac{1}{15}+\frac{1}{15}=\frac{2}{15}.$$

Make the observation that

$$\frac{2}{3} \times \frac{1}{5} = \frac{2 \cdot 1}{3 \cdot 5} = \frac{2}{15}.$$

With this observation in mind we can generalize the multiplication of fractions as follows.

Multiplication of Fractions

The product of $\frac{a}{b}$ and $\frac{c}{d}$ is the fraction $\frac{a \cdot c}{b \cdot d}$.

EXAMPLE 1 Perform the indicated operation(s): $\frac{3}{4} \cdot \frac{5}{7}$.

SOLUTION: Following the algorithm given above,

$$\frac{3}{4} \cdot \frac{5}{7} = \frac{3 \cdot 5}{4 \cdot 7} = \frac{15}{28}.$$

The product of two fractions is a fraction in which the numerator is the product of the original numerators and the denominator is the product of the original denominators.

EXAMPLE 2 Perform the indicated operation(s): $\frac{8}{3}\left(2\frac{1}{4}\right)$.

SOLUTION: Changing the mixed number to an improper fraction and following the algorithm given above we get

$$\frac{8}{3} \cdot \frac{9}{4} = \frac{8 \cdot 9}{3 \cdot 4}.$$

Simplifying before performing the multiplication gives

$$\frac{8 \cdot 9}{3 \cdot 4} = \frac{4 \cdot 2 \cdot 3 \cdot 3}{3 \cdot 4} = \frac{\cancel{4} \cdot 2 \cdot \cancel{3} \cdot 3}{\cancel{3} \cdot \cancel{4}} = 2 \cdot 3$$

Therefore, $\frac{8}{3} \cdot \frac{9}{4} = 6$.

Avoiding Mistakes

Do not confuse addition of fractions with multiplication of fractions. When multiplying fractions we do not need a common denominator, and we multiply both the numerators and the denominators.

Notice the difference between Example 1 and Example 2. In Example 1 the numerators and denominators did not share any common factors while in Example 2 they did.

EXAMPLE 3 Perform the indicated operation(s): $\left(\frac{2}{5}\right)\left(-\frac{3}{10}\right)\left(\frac{1}{4}\right)$.

SOLUTION: According the Order of Operations we perform multiplication from left to right.

$$\left(\frac{2}{5}\right)\left(-\frac{3}{10}\right)\left(\frac{1}{4}\right) = \left[\left(\frac{2}{5}\right)\left(-\frac{3}{10}\right)\right]\left(\frac{1}{4}\right)$$

$$= \left[\frac{2(-3)}{5\cdot 10}\right]\left(\frac{1}{4}\right)$$

$$= \left[\frac{\cancel{2}(-3)}{5\cdot \cancel{2}\cdot 5}\right]\left(\frac{1}{4}\right)$$

$$= \frac{-3}{25}\left(\frac{1}{4}\right)$$

$$= \frac{-3\cdot 1}{25\cdot 4}$$

$$= -\frac{3}{100}$$

Therefore, $\left(\frac{2}{5}\right)\left(-\frac{3}{10}\right)\left(\frac{1}{4}\right) = -\frac{3}{100}$.

4.4.2 Division of Fractions

The Reciprocal of a Fraction

The Reciprocal of a Fraction
The reciprocal of the fraction $\frac{a}{b}$ is the fraction $\frac{b}{a}$.

This definition covers the reciprocals of integers as well. This follows because any integer is also a fraction with a denominator of 1. As an example, the reciprocal of

$$5 = \frac{5}{1}$$

is the fraction

$$\frac{1}{5}.$$

An important property of reciprocals is that their product is always 1. That is, the product of the fraction $\frac{a}{b}$ and its reciprocal is

$$\frac{a}{b} \cdot \frac{b}{a} = \frac{a \cdot b}{b \cdot a} = 1$$

since anything divided by itself is 1.

EXAMPLE 1 Find the reciprocal of $\frac{3}{8}$.

SOLUTION: The reciprocal is

$$\frac{8}{3}$$

We can verify this by noting that

$$\frac{3}{8} \cdot \frac{8}{3} = \frac{24}{24} = 1.$$

EXAMPLE 2 Find the reciprocal of $-\frac{9}{5}$.

SOLUTION: The reciprocal is

$$-\frac{5}{9}$$

We can verify this by noting that

$$-\frac{9}{5}\left(-\frac{5}{9}\right) = \frac{9 \cdot 5}{5 \cdot 9} = \frac{45}{45} = 1.$$

Division of Fractions

Consider the problem of dividing $\frac{7}{3}$ by $\frac{4}{5}$. That is, we want to calculate

$$\frac{7}{3} \div \frac{4}{5} = \frac{\frac{7}{3}}{\frac{4}{5}}.$$

We handle problems like this by taking advantage of two properties of multiplying and dividing Real Numbers. The first property is that any number times 1 is itself. This means that we can multiply any number by 1 without changing it. The second property is that anything divided by itself is 1. The second property really says, for this example in particular, that

$\frac{\frac{5}{4}}{\frac{5}{4}} = 1$. Note that this is the reciprocal of the denominator divided by itself.

Since this equals 1, we can multiply any number by it without changing that number. In particular,

$$\frac{\frac{7}{3}}{\frac{4}{5}} = \frac{\frac{7}{3}}{\frac{4}{5}} \cdot \frac{\frac{5}{4}}{\frac{5}{4}}$$

Performing the multiplication in the numerator and the denominator we get

$$\frac{\frac{7}{3}}{\frac{4}{5}} \cdot \frac{\frac{5}{4}}{\frac{5}{4}} = \frac{\frac{7}{3} \cdot \frac{5}{4}}{\frac{4}{5} \cdot \frac{5}{4}}$$

In the denominator we have a fraction multiplied by its reciprocal so the product in the denominator is just 1.

$$\frac{\frac{7}{3} \cdot \frac{5}{4}}{\frac{4}{5} \cdot \frac{5}{4}} = \frac{\frac{7 \cdot 5}{3 \cdot 4}}{1}$$

Since anything divided by itself is 1 we are left with

$$\frac{7 \cdot 5}{3 \cdot 4}.$$

Finishing the multiplication we get $\frac{35}{12}$.

Therefore,

$$\frac{\frac{7}{3}}{\frac{4}{5}} = \frac{7}{3} \cdot \frac{5}{4} = \frac{35}{12}.$$

The rule for dividing fractions follows this example.

Division of Fractions

The quotient of $\frac{a}{b}$ and $\frac{c}{d}$ is

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c}$$

The rule states that to divide by a fraction, we multiply by its reciprocal.

EXAMPLE 1 Perform the indicated operation(s): $\frac{5}{4} \div \frac{1}{3}$.

SOLUTION: To calculate

$$\frac{\frac{5}{4}}{\frac{1}{3}}$$

we need to multiply the numerator and denominator by the reciprocal of the divisor.

The divisor in this instance is $\frac{1}{3}$. The reciprocal of the divisor is then $\frac{3}{1} = 3$.

So,

$$\frac{\frac{5}{4}}{\frac{1}{3}} = \frac{\frac{5}{4} \cdot \frac{3}{1}}{\frac{1}{3} \cdot \frac{3}{1}} = \frac{\frac{5 \cdot 3}{4 \cdot 1}}{1} = \frac{15}{4}.$$

We can run through this process every time we need to divide by a fraction. However, according the rule, to divide by a fraction, multiply by its reciprocal.

That is, since we are dividing by $\frac{1}{3}$ we multiply by its reciprocal. That is,

$$\frac{\frac{5}{4}}{\frac{1}{3}} = \frac{5}{4} \cdot \frac{3}{1} = \frac{15}{4}.$$

EXAMPLE 2 Perform the indicated operation(s): $\frac{3}{7} \div \frac{2}{5}$.

SOLUTION: To perform the division by $\frac{2}{5}$ we multiply by its reciprocal. Therefore,

$$\frac{\frac{3}{7}}{\frac{2}{5}} = \frac{3}{7} \cdot \frac{5}{2} = \frac{15}{14}.$$

EXAMPLE 3 Perform the indicated operation(s): $-\frac{1}{6} \div \frac{4}{9}$.

SOLUTION: To perform the division by $\frac{4}{9}$ we multiply by its reciprocal. Therefore,

$$\frac{-\frac{1}{6}}{\frac{4}{9}} = \frac{-1}{6} \cdot \frac{9}{4}$$

$$= \frac{-1 \cdot 3 \cdot 3}{2 \cdot 3 \cdot 2 \cdot 2}$$

$$= -\frac{1 \cdot \cancel{3} \cdot 3}{2 \cdot \cancel{3} \cdot 2 \cdot 2}$$

$$= -\frac{3}{8}.$$

EXAMPLE 4 Perform the indicated operation(s): $-\frac{7}{4} \div (-14)$.

SOLUTION: To perform the division by -14 we multiply by its reciprocal. That is,

$$\frac{-\frac{7}{4}}{-14} = \left(-\frac{7}{4}\right)\left(\frac{1}{-14}\right)$$

$$= \frac{-7 \cdot 1}{4 \cdot (-14)}$$

$$= \frac{-7 \cdot 1}{4(-7)(2)}$$

$$= \frac{\cancel{-7} \cdot 1}{4(\cancel{-7})(2)}$$

$$= \frac{1}{8}.$$

EXAMPLE 5 Perform the indicated operation(s): $\frac{2}{9} \cdot \frac{4}{5} \div \left(3\frac{3}{4}\right)$.

SOLUTION: According to the Order of Operations we perform multiplication and division as they occur from left to right. Therefore, after changing the mixed number to an improper fraction, we have

$$\frac{2}{9} \cdot \frac{4}{5} \div \left(3\frac{3}{4}\right) = \frac{2}{9} \cdot \frac{4}{5} \div \frac{15}{4}$$

$$= \left(\frac{2}{9} \cdot \frac{4}{5}\right) \div \frac{15}{4}$$

$$= \left(\frac{2 \cdot 4}{9 \cdot 5}\right) \div \frac{15}{4}$$

$$= \frac{8}{45} \div \frac{15}{4}$$

$$= \frac{8}{45} \cdot \frac{4}{15}$$

$$= \frac{8 \cdot 4}{45 \cdot 15}$$

$$= \frac{32}{675}.$$

EXAMPLE 6 Perform the indicated operation(s): $-3 + \frac{5}{3} \cdot \frac{1}{4} \div 3 - \frac{3}{2} \div 6.$

SOLUTION: According to the Order of Operations we perform multiplication and division as they occur from left to right and then addition and subtraction as they occur from left to right. Performing the multiplication and division from left to right we have

$$-3 + \frac{5}{3} \cdot \frac{1}{4} \div 3 - \frac{3}{2} \div 6 = -3 + \frac{5 \cdot 1}{3 \cdot 4} \div 3 - \frac{3}{2} \div 6$$

$$= -3 + \frac{5}{12} \div 3 - \frac{3}{2} \div 6.$$

$$= -3 + \frac{5}{12} \cdot \frac{1}{3} - \frac{3}{2} \cdot \frac{1}{6}$$

$$= -3 + \frac{5 \cdot 1}{12 \cdot 3} - \frac{3 \cdot 1}{2 \cdot 6}$$

$$= -3 + \frac{5}{36} - \frac{1}{4}.$$

To perform the addition and subtraction note that the LCD = 36.

$$= \frac{(-3)(36)}{36} + \frac{5}{36} - \frac{1 \cdot 9}{4 \cdot 9}$$

$$= \frac{-108+5-9}{36}$$

$$= \frac{-112}{36}.$$

$$= -\frac{4 \cdot 28}{4 \cdot 9}$$

$$= -\frac{28}{9}.$$

EXAMPLE 7 Perform the indicated operation(s): $-2\frac{3}{5}+3 \div \frac{12}{5}-\frac{1}{2}\left(-1\frac{2}{3}\right)$.

SOLUTION: Before applying the Order of Operations we write the mixed numbers as improper fractions.

$$\Rightarrow \ -2\frac{3}{5}+3 \div \frac{12}{5}-\frac{1}{2}\left(-1\frac{2}{3}\right) = -\frac{13}{5}+3 \div \frac{12}{5}-\frac{1}{2}\left(-\frac{5}{3}\right)$$

We can now perform multiplication and division as they occur from left to right.

$$\Rightarrow \ -\frac{13}{5}+3 \cdot \frac{5}{12}-\frac{1}{2}\left(-\frac{5}{3}\right)$$

$$= -\frac{13}{5}+\frac{3 \cdot 5}{12}-\frac{1}{2}\left(-\frac{5}{3}\right)$$

$$= -\frac{13}{5}+\frac{5}{4}-\frac{1}{2}\left(-\frac{5}{3}\right)$$

$$= -\frac{13}{5}+\frac{5}{4}+\frac{1 \cdot 5}{2 \cdot 3}$$

$$= -\frac{13}{5}+\frac{5}{4}+\frac{5}{6}$$

We can now perform the addition and subtraction from left to right.

$$= \left(-\frac{13}{5}\right)\left(\frac{12}{12}\right) + \frac{5}{4}\left(\frac{15}{15}\right) + \frac{5}{6}\left(\frac{10}{10}\right)$$

$$= -\frac{156}{60} + \frac{75}{60} + \frac{50}{60}$$

$$= -\frac{31}{60}.$$

Section 4.4 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 10, perform the indicated multiplication.

1) $\frac{4}{5} \cdot 3$ 2) $-6 \cdot \frac{2}{3}$ 3) $-\frac{7}{2} \cdot (-2)$ 4) $\frac{5}{3} \cdot (-7)$ 5) $\left(\frac{7}{5}\right)\left(\frac{1}{3}\right)$

6) $-\frac{2}{9} \cdot \frac{6}{5}$ 7) $\frac{5}{4} \cdot \frac{3}{5}$ 8) $\left(\frac{3}{2}\right)\left(\frac{2}{3}\right)$ 9) $\frac{4}{3} \cdot -5$ 10) $\frac{6}{7} \cdot \frac{14}{3}$

For problems 11 - 15, find the reciprocal of the given number.

11) -5 12) $\frac{4}{3}$ 13) $\frac{7}{6}$ 14) $-\frac{11}{8}$ 15) $-\frac{1}{3}$

For problems 16 - 25, perform the indicated division.

16) $\frac{3}{5} \div \frac{1}{2}$ 17) $\frac{9}{5} \div \frac{3}{7}$ 18) $-\frac{1}{3} \div -\frac{1}{4}$ 19) $2 \div \frac{3}{2}$ 20) $\frac{5}{2} \div 4$

21) $3 \div -\frac{1}{4}$ 22) $\frac{9}{5} \div \frac{3}{10}$ 23) $-\frac{5}{3} \div \frac{1}{3}$ 24) $6 \div \frac{3}{7}$ 25) $\frac{11}{5} \div 3$

For problems 26 - 30, perform the indicated operations.

26) $\frac{3}{2} \cdot \frac{5}{3} \div 2$ 27) $5 + \frac{1}{5} \div \frac{2}{3} - \frac{1}{6}$ 28) $-\frac{3}{7} + \frac{3}{7} \div \frac{1}{14} + \frac{12}{5} \cdot \frac{10}{3}$

29) $\left(4\frac{1}{2}\right) \div \frac{5}{3} \div \left(1\frac{2}{5}\right)$ 30) $\left(-2\frac{2}{5}\right)\left(\frac{7}{3}\right) + 6 \div \frac{3}{5}$ 31) $\left(3\frac{1}{3}\right) \div \left(-2\frac{3}{5}\right)\left(\frac{13}{5}\right)$

32) $2 + 3\frac{1}{6}\left(\frac{2}{5}\right) - \frac{1}{2}$

Review. Perform the indicated operation(s).

33) $2\frac{13}{16}+1\frac{3}{4}$

34) $6\frac{4}{5}-3\frac{16}{25}$

35) $17\frac{5}{12}-15\frac{7}{20}$

Chapter 4
Section 4.4 – Multiplication and Division of Fractions

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

1) _____

2) _____

3) _____

4) _____

5) _____

6) _____

7) _____

8) _____

9) _____

10) _____

11) _____

12) _____

13) _____

14) _____

15) _____

16) _____

17) _____

18) _____

19) _____

20) _____

21) _____

22) _____

23) _____

24) _____

25) _____

26) _____

27) _____

28) _____

29) _____

30) _____

31) _____

32) _____

33) _____

34) _____

35) _____

WORKING WITH DECIIMALS

CHAPTER

5

CHAPTER OBJECTIVES

5.1 Introduction to Decimals: Notation and Rounding

5.2 Addition and Subtraction of Decimals

5.3 Converting Decimals to Fractions and Fractions to Decimals

5.4 Multiplication and Division with Decimals

n - Factorial

In mathematics, the factorial of a non-negative integer n, denoted by n!, is the product of all positive integers less than or equal to n. For example, the value of 5! is 5 x 4 x 3 x 2 x 1 = 120. The factorial operation is encountered in many areas of mathematics, notably in combinatorics, algebra, and mathematical analysis. Its most basic occurrence is the fact that there are n! ways to arrange n distinct objects into a sequence. This fact was known at least as early as the 12th century to Indian scholars. Fabian Stedman in 1677 described factorials as applied to change ringing. After describing a recursive approach, Stedman gives a statement of a factorial: Now the nature of these methods is such, that the changes on one number comprehends the changes on all lesser numbers ... The notation n! was introduced by Christian Kramp in 1808.

5.1 Introduction to Decimals: Notation and Rounding

5.1.1 Decimal Notation

Recall from chapter 1 that each digit of a whole number has a place value. We used the number $987,654,321$ along with the following chart to illustrate this idea.

Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
9	8	7,	6	5	4,	3	2	1

From the chart we see that the digit 5 is in the ten thousands place. This meant that its contribution to the number was five $10,000$'s or $50,000$. Further along in chapter 1 we introduced natural number exponents and powers of 10 so that ten thousand could be represented as 10^4 and so the digit 5's contribution to the number could be written as $5 \cdot 10^4$.

In fact, we can rewrite the entire chart with powers of 10 as shown here.

10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0
9	8	7,	6	5	4,	3	2	1

One thing to point out here is that as we move to the left from the ones position, the digit's place value increases by a power of 10. This is actually the defining difference between integers and **decimals**. Decimals are numbers that contain an integer part along with a fractional part. The integer portion is separated from the fractional portion with a **decimal point**.

The decimal point is located directly to the right of the ones place and since we have worked with integers up until now there hasn't been any reason to include it. That is, we could have written the integers 4, 29, and -467 with the decimal point as

$$4.,\ 29.,\ \text{and}\ -467.$$

but there was no reason to as pure integer values do not contain a fractional component. Some examples of decimals are

$$1.35,\ -5.7008,\ \text{and}\ 13.2.$$

For 1.35, the integer component is 1 and the fractional component is .35. For -5.7008, the integer component is -5 and the fractional component is .7008.

With whole numbers, as we move to the left from the ones place, the place value increases by a power of 10. With the fractional component of decimals, as we move to the right from the ones place, the place value **decreases** by a power of 10. To illustrate this we can create a chart for the number 89.1234567.

10^{1}	10^{0}	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
8	9.	1	2	3	4	5	6	7

From the chart, the digit 1 has a value of 10^{-1}, the digit 2 has a value of 10^{-2}, the digit 3 has a value of 10^{-3}, etc. The terminology for these place values is as follows.

- 10^{-1} - ten**ths**
- 10^{-2} - hundred**ths**
- 10^{-3} - thousand**ths**
- 10^{-4} - ten thousand**ths**
- 10^{-5} - hundred thousand**ths**

$\vdots$

It is important to know that when we refer to the tenths place, the place value of 10^{-1} really holds the *fractional* value of $\dfrac{1}{10^{1}} = \dfrac{1}{10}$.

The hundredths, or 10^{-2} place, really holds the *fractional* value of $\frac{1}{10^2} = \frac{1}{100}$.

The thousandths, or 10^{-3} place, holds the *fractional* value of $\frac{1}{10^3} = \frac{1}{1000}$.

In general, the place value of 10^{-n} holds the fractional value of $\frac{1}{10^n}$.

Again, just as we do with integers, it is important to know how to communicate the value of a decimal. That is, we know that the integer $5,203$ is stated verbally as "five thousand two hundred and three."

We always state integer values starting with the largest place value of the number.

When we can, we always state the value of a decimal based on the smallest place value. For example,

1.35 is stated as "one **and** thirty five hundredths"

because the smallest place value, the place value furthest to the right, of the decimal is held by the 5 in the hundredths position. Note we use "**and**" to define the position of the decimal point.

-2.109 is stated as "negative two **and** one hundred and nine thousandths"

because the smallest place value of the decimal is held by the 9 in the thousandths position.

Another way to state the value of a decimal is to simply state the numbers following the decimal. That is,

1.35 can be stated as "one **point** three five."

-2.109 can be stated as "negative two **point** one, zero, nine."

Note that here we use "**point**" to define the position of the decimal point.

Either way we state the value of a decimal, we always begin by providing the value of the integer part and then we provide the value of the fractional part.

EXAMPLE 1 State the value of the decimal 2.034 in two different ways.

SOLUTION: For the number 2.034, the smallest place value is in the thousandths. So one way to state the decimal is

two and thirty four thousandths.

The other way to state the decimal is

two point zero, three, four.

We can even combine the two versions and state the decimal as

two point thirty four thousandths.

EXAMPLE 2 State the value of the decimal -0.5208 in two different ways.

SOLUTION: For the number -0.5208, the smallest place value is in the ten-thousandths. So one way to state the decimal is

negative zero and five thousand two hundred and eight ten-thousandths.

Because the integer value if zero we could say

negative five thousand two hundred and eight ten-thousandths.

The other way to state the decimal is

negative zero point five, two, zero, eight.

Similar to what we did in section 1.1.1 with whole numbers, we can write decimals in expanded form. This will help us add, subtract, multiply, and divide decimals.

We will use the fact that

- tenths = $10^{-1} = \frac{1}{10}$
- hundredths = $10^{-2} = \frac{1}{100}$
- thousandths = $10^{-3} = \frac{1}{1,000}$
- ten thousandths = $10^{-4} = \frac{1}{10,000}$

$\vdots$

to write decimals as fractions and to write decimals in expanded form.

EXAMPLE 3 Write the decimal 8.049 in expanded form.

SOLUTION: The digits and place holders in the decimal 8.049 are as follows.

- ones - $8 = 8 \cdot 10^0$
- tenths - $0 = 0 \cdot \frac{1}{10}$
- hundredths - $4 = 4 \cdot \frac{1}{100}$
- thousandths - $9 = 9 \cdot \frac{1}{1,000}$

So, 8.049 can be written in expanded form as

$$8 + 4 \cdot \frac{1}{100} + 9 \cdot \frac{1}{1,000}.$$

Make the observation that because we know this is really

$$8 + \frac{4}{100} + \frac{9}{1,000} \quad \text{we can simplify to get} \quad 8 + \frac{1}{25} + \frac{9}{1,000}.$$

However, the expanded form should be in terms of powers of 10.

EXAMPLE 4 Write the decimal 350.1963 in expanded form.

SOLUTION: The digits and place holders in the decimal 350.1963 are as follows.

- hundreds - $3 = 3 \cdot 10^2$
- tens - $5 = 5 \cdot 10^1$
- ones - $0 = 0 \cdot 10^0$
- tenths - $1 = 1 \cdot \frac{1}{10}$
- hundredths - $9 = 9 \cdot \frac{1}{100}$
- thousandths - $6 = 6 \cdot \frac{1}{1,000}$
- ten-thousandths - $3 = 3 \cdot \frac{1}{10,000}$

The decimal in expanded form is then

$$3 \cdot 100 + 5 \cdot 10 + 1 \cdot \frac{1}{10} + 9 \cdot \frac{1}{100} + 6 \cdot \frac{1}{1,000} + 3 \cdot \frac{1}{10,000}.$$

5.1.2 Rounding Decimals

In chapter 1 we learned how to round integers. We will follow very similar rules to round decimals to an indicated position or place value.

Rounding Decimals

Identify the location of the requested place value to which you want to round.

- If the digit to the right of the requested place value is between 0 and 4, remove all of the digits to the right of the requested place value.
- If the digit to the right of the requested place value is between 5 and 9, add one to the digit in the requested place value and remove all of the digits to the right of the requested place value.

EXAMPLE 1 Round 4.3624 to the nearest tenth.

SOLUTION: The digit in the tenth place is 3. The digit to the right of it, the digit in the hundredths place, is 6. Because 6 is between 5 and 9, increase the digit in the tenths place by one and remove all of the digits to the right of the tenths place.

4.3624 rounded to the nearest tenth is 4.4.

EXAMPLE 2 Round 4.3624 to the nearest thousandth.

SOLUTION: The digit in the thousandth place is 2. The digit to the right of it, the digit in the ten-thousandths place, is 4. Because 4 is between 0 and 4 we need to remove all digits to the right of the thousandth place.

4.3624 rounded to the nearest thousandth is 4.362.

EXAMPLE 3 Round -12.7033 to the nearest ones.

SOLUTION: The digit in the ones place is 2. The digit to the right of the ones place, the digit in the tenths place, is 7. Because 7 is between 5 and 9 we need to add one to the digit in the ones place and remove all digits to the right of it.

-12.7033 rounded to the nearest ones is -13.

Section 5.1 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 5, identify the integer portion and the fractional portion of the given number.

1) 4.108 2) -12.56 3) 562.458 4) -20 5) 0.009

For problems 6 - 10, identify the place value of the underlined digit.

6) $4.3\underline{2}0$ 7) $1\underline{0},300$ 8) $0.10\underline{3}2$ 9) $231.581\underline{6}$ 10) $100.\underline{3}12$

For problems 11 - 15, state the value of the decimal.

11) 1.3 12) 0.041 13) 300.51 14) 20.2513 15) -11.0056

For problems 16 - 20, translate the phrase into a decimal.

16) Twenty and two tenths

17) One and one hundredth

18) Negative eleven thousandths

19) Five thousand, three hundred and eleven thousandths

20) Six hundred twenty three ten thousandths

For problems 21 - 24, round 72.315374 to the indicated place value.

21) Hundredths 22) Whole Number 23) Thousandths 24) Tenths

For problems 25 - 28, write the given decimal in expanded form.

25) 3.45 26) 2.0401 27) 0.166 28) 11.123

Chapter 5
Section 5.1 – Introduction to Decimals: Notation and Rounding

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

1) Integer: _____ Fractional: _____

2) Integer: _____ Fractional: _____

3) Integer: _____ Fractional: _____

4) Integer: _____ Fractional: _____

5) Integer: _____ Fractional: _____

6) _____

7) _____

8) _____

9) _____

10) _____

11) _____

12) _____

13) _____

14) _____

15) _____

16) _____

17) _____

18) _____

19) _____

20) _____

21) _____

22) _____

23) _____

24) _____

25) _____

26) _____

27) _____

28) _____

5.2 Addition and Subtraction of Decimals

5.2.1 Addition

Just as we do when adding integers, we will line up the decimals vertically and add the digits in corresponding place values.

Make the observation that to visually line up decimals of different lengths, we can add zeroes to the right of a decimal without changing its value. That is,

$$1.5,\ 1.50,\ \text{and}\ 1.500$$

are all the same decimal.

EXAMPLE 1 Perform the indicated operation. $3.56 + 10.703$.

SOLUTION: Write the addition vertically making sure to line up the decimal point.

$$\begin{array}{r} 3.560 \\ +\ 10.703 \\ \hline \end{array}$$

Notice that 3.56 goes out to the hundredths while 10.703 goes to the thousandths. To visually keep the decimals lined up we added a 0 to the thousandths place of 3.56. This does not change its value.

To perform the addition we simply add as usual but now we must make sure to keep the position of the decimal point in the correct spot. The correct spot is between the ones place and the tenths place.

$$\begin{array}{r} {\scriptstyle 1} \\ 3.560 \\ +\ 10.703 \\ \hline 14.263 \end{array}$$

Notice that during the addition process, adding the digits in the tenths place gave us

$$5 \cdot \frac{1}{10} + 7 \cdot \frac{1}{10} = \frac{5+7}{10}$$

$$= \frac{12}{10}$$

$$= \frac{10+2}{10}$$

$$= \frac{10}{10} + \frac{2}{10}$$

$$= 1 + \frac{2}{10}$$

and we had to carry an additional "one" into the ones place.

EXAMPLE 2 Perform the indicated operation. $15.0608 + 9.8187$.

SOLUTION: Write the number vertically making sure to line up the decimal point.

$$\begin{array}{r} 1\ 5.0\ 6\ 0\ 8 \\ +\quad 9.8\ 1\ 8\ 7 \\ \hline \end{array}$$

Perform the addition.

$$\begin{array}{r} {\scriptstyle 1\qquad\quad 1} \\ 1\ 5.0\ 6\ 0\ 8 \\ +\quad 9.8\ 1\ 8\ 7 \\ \hline 2\ 4.8\ 7\ 9\ 5 \end{array}$$

Therefore, $15.0608 + 9.8187 = 24.8795$.

5.2.2 Subtraction

The subtraction of decimals follows the same rules as subtraction of integers.

EXAMPLE 1 Perform the indicated operation. $4.026 - 3.817$

SOLUTION: Performing subtraction with integers relied on our ability to determine which integer was larger. We then subtracted the smaller integer from the larger integer and then gave the result the sign of the larger integer.

Determining which decimal is larger begins with the integer portion. For this problem, the integer part of 4.026 is larger than the integer part of 3.817 and so

$$4.026 \geq 3.817.$$

This means we need to subtract 3.817 from 4.026 and make the result positive.

So for this problem we have the following.

$$\begin{array}{r} 4.026 \\ -\ 3.817 \\ \hline \end{array}$$

Notice that we will have to "borrow" from some place values so that we can perform the subtraction.

In this problem, to perform the subtraction in the thousandths place we will have to borrow from the hundredths. To perform the subtraction in the tenths place we will have to borrow from the ones.

$$\begin{array}{r} 3\ \ 10\ \ 1\ \ 16 \\ \not{4}.\not{0}\ \not{2}\ \not{6} \\ -\ 3.817 \\ \hline 0.209 \end{array}$$

Therefore, $4.026 - 3.817 = 0.209$.

EXAMPLE 2 Perform the indicated operation. $9.025 - 12.1562$

SOLUTION: For this problem, the integer part of 12.1562 is larger than the integer part of 9.025 and so

$$12.1562 \geq 9.025$$

and we subtract 9.025 from 12.1562 and give make the result negative.

$$\begin{array}{r} 1\ \ 2.\ 1\ \ 5\ \ 6\ \ 2 \\ -\quad 9.\ 0\ \ 2\ \ 5\ \ 0 \\ \hline \end{array}$$

Performing the subtraction gives us the following.

$$\begin{array}{r} \color{blue}{0}\ \ \color{blue}{12} \\ \not{1}\ \not{2}.\ 1\ \ 5\ \ 6\ \ 2 \\ -\quad 9.\ 0\ \ 2\ \ 5\ \ 0 \\ \hline 3.\ 1\ \ 3\ \ 1\ \ 2 \end{array}$$

Therefore, $9.025 - 12.1562 = -3.1312$.

5.2.3 Addition and Subtraction

EXAMPLE 1 Perform the indicated operation(s). $-6.12 + 3.7 - 5.09$

SOLUTION: The Order of Operations says that we must perform addition and subtraction as they occur from left to right. Here this means that we need to first calculate $-6.12 + 3.7$. Because $6.12 \geq 3.7$, we subtract 3.7 from 6.12 and make the result negative.

$$\begin{array}{r} \color{blue}{5}\ \ \color{blue}{11} \\ \not{6}.\ \not{1}\ \ 2 \\ -\ 3.\ 7\ \ 0 \\ \hline 2.\ 4\ \ 2 \end{array}$$

We can then write

$$-6.12+3.7-5.09=-2.42-5.09.$$

Since both 2.42 and 5.09 are negative we add them and make the result negative.

$$\begin{array}{r} {\scriptstyle 1} \\ 2.42 \\ +\ 5.09 \\ \hline 7.51 \end{array}$$

Therefore, $-6.12+3.7-5.09=-7.51$.

EXAMPLE 2 Perform the indicated operation(s). $0.805-0.724+1.233$

SOLUTION: Since we must perform addition and subtraction as they occur from left to right we begin by calculating $0.805-0.724$. The integer part of 0.805 and 0.724 is the same so we need to move to the tenths to determine which number is larger. The digit in the tenths place of 0.805 is 8 while the digit in the tenths place of 0.724 is 7. This means that $0.805 \geq 0.724$ so we subtract 0.724 from 0.805 and make the result positive.

$$\begin{array}{r} {\scriptstyle 7\ 10} \\ 0.\not{8}\ \not{0}\ 5 \\ -\ 0.7\ 2\ 4 \\ \hline 0.0\ 8\ 1 \end{array}$$

At this point we have $0.805-0.724+1.233=0.081+1.233$. To finish the problem we need to calculate $0.081+1.233$.

$$\begin{array}{r} {\scriptstyle 1} \\ 1.2\ 3\ 3 \\ +\ 0.0\ 8\ 1 \\ \hline 1.3\ 1\ 4 \end{array}$$

$0.805-0.724+1.233=1.314$.

EXAMPLE 3 Perform the indicated operation(s). $4.30678 - 4.30682 - 2.5006$

SOLUTION: The numbers 4.30678 and 4.30682 have the same digits in each place value until we get to the ten thousandths place. Because $8 \geq 7$, $4.30682 \geq 4.30678$ so we must subtract 4.30678 from 4.30682 and make the result negative.

$$\begin{array}{r} \ \ 7\ \ 12 \\ 4.3\ 0\ 6\ \cancel{8}\ \cancel{2} \\ -\ 4.3\ 0\ 6\ 7\ 8 \\ \hline 0.0\ 0\ 0\ 0\ 4 \end{array}$$

We now have

$$4.30678 - 4.30682 - 2.5006 = -0.00004 - 2.5006.$$

Both values are negative so we can add them and make the result negative.

$$\begin{array}{r} 2.5\ 0\ 0\ 6\ 0 \\ +\ 0.0\ 0\ 0\ 0\ 4 \\ \hline 2.5\ 0\ 0\ 6\ 4 \end{array}$$

Therefore, $4.30678 - 4.30682 - 2.5006 = -2.50064$.

Section 5.2 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 10, perform the addition.

1) $1.32 + 2.41$
2) $10.205 + 3.72$
3) $3.53 + 6.4007$
4) $0.711 + 5.183$
5) $13.0507 + 4.332$
6) $3.145 + 4.591$
7) $13.021 + 4.389$
8) $23.79 + 5.461$
9) $1.0405 + 5.692$
10) $3.1 + 5.46$

For problems 11 - 20, perform the subtraction.

11) $5.4 - 1.3$
12) $10.435 - 2.124$
13) $2.975 - 0.52$
14) $12.3964 - 3.154$
15) $3.56 - 1.031$
16) $6.3401 - 46.324$
17) $7.3512 - 19.48$
18) $1.062 - 4.5$
19) $4.5007 - 4.5338$
20) $7.08 - 3.5222$

For problems 21 - 25, perform the indicated operations.

21) $2.31 + 5.47 - 1.06$
22) $0.34 - 9.587 + 5.7012$
23) $5.319 + 6.732 - 9.41$
24) $6.38 - 7.1581 + 14.719$
25) $35.6 - 26.715 - 54.132$

Review. Perform the indicated operation(s). Write your answer as a rational number.

26) $\frac{1}{10} + \frac{2}{5}$
27) $\frac{3}{8} - 1\frac{4}{9}$
28) $0 \div 5\frac{1}{3}$
29) $\left(4\frac{2}{7}\right) \div \left[1\frac{1}{3}\right]$
30) $\left(-\frac{3}{7}\right)\left(-\frac{8}{5}\right)$
31) $\frac{35}{18} \cdot \frac{27}{14}$

Chapter 5
Section 5.2 – Addition and Subtraction of Decimals

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

1) _____

2) _____

3) _____

4) _____

5) _____

6) _____

7) _____

8) _____

9) _____

10) _____

11) _____

12) _____

13) _____

14) _____

15) _____

16) _____

17) _____

18) _____

19) _____

20) _____

21) _____

22) _____

23) _____

24) _____

25) _____

26) _____

27) _____

28) _____

29) _____

30) _____

31) _____

5.3 Converting Decimals to Fractions and Fractions to Decimals

5.3.1 Converting Decimals to Fractions

To convert a decimal to a fraction we separate the integer portion from the fractional component, write the fractional component in terms of the lowest place value in the decimal, and then add the integer portion with the fractional component.

As an example, write 2.45 as a fraction. Because 2.45 is "two and forty five hundredths" we can write it as

$$2.45 = 2 + \frac{45}{100}.$$

We can then simplify the fraction and add it to the integer 2. Therefore,

$$2.45 = 2 + \frac{9}{20}$$

$$= \frac{40}{20} + \frac{9}{20}$$

$$= \frac{49}{20}.$$

EXAMPLE 1 Write the decimal 4.52 as a fraction.

SOLUTION: 4.52 is "four and fifty two hundredths" so we can write it as

$$4.52 = 4 + \frac{52}{100}.$$

Simplifying the fraction and performing the addition gives

$$= 4 + \frac{13}{25}$$

$$= \frac{100}{25} + \frac{13}{25}$$

$$= \frac{113}{25}.$$

EXAMPLE 2 Write the decimal 1.0285 as a fraction.

SOLUTION: Since 1.0285 can be interpreted as "one and two hundred and eighty five ten-thousandths",

$$1.0285 = 1 + \frac{285}{10,000}$$

$$= 1 + \frac{5 \cdot 57}{5 \cdot 2,000}$$

$$= 1 + \frac{57}{2,000}.$$

$$= \frac{2,000}{2,000} + \frac{57}{2000}$$

$$= \frac{2,057}{2,000}.$$

5.3.2 Converting Fractions to Decimals

Recall that the fraction $\frac{a}{b}$ means $a \div b$ so to convert a fraction to a decimal we need to calculate the division.

In the past, division resulted in integers or mixed numbers. We now allow the quotients to be decimals.

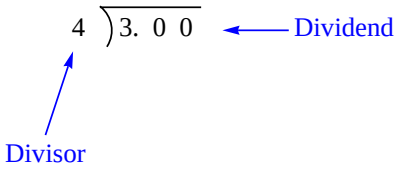
EXAMPLE 1 Write the fraction $\frac{3}{4}$ as a decimal.

SOLUTION: To write a fraction as a decimal, recall that

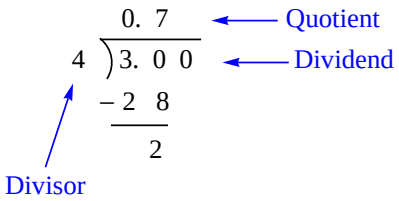
$$\frac{a}{b} \text{ means } a \div b.$$

This implies that if we wish to write a fraction as a decimal we should perform the division. For this problem, we need to divide 3 by 4.

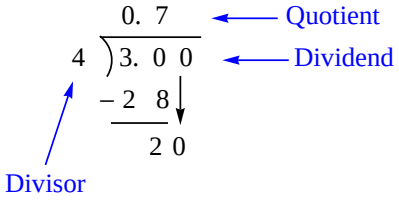
Because 4 does not "go into" 3, we need to rewrite the integer 3 as a decimal. In other words, write 3 as 3.00 and then perform the division.



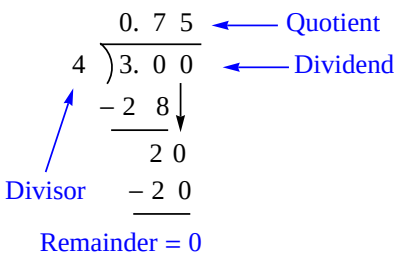
1. Set up the division as usual.
2. The divisor is 4 and the dividend is 3.



3. 4 goes into 3 zero times but it goes into 30, 7 times.
4. Subtract $4 \cdot 7$ from 30: $30 - 4 \cdot 7 = 2$.
5. Notice that the decimal point in the quotient lines up with the decimal point in the dividend.



6. Bring the next digit in the dividend down.



7. 4 goes into 20, 5 times.
8. Subtract $4 \cdot 5$ from 20: $20 - 4 \cdot 5 = 0$.
9. Since the remainder is zero, and there are no more digits in the dividend to bring down, the division is complete.

This says that

$$\frac{3}{4} = 0.75.$$

We can check this by noting that 0.75 is "seventy five hundredths" and

$$\frac{75}{100} = \frac{25 \cdot 3}{25 \cdot 4} = \frac{3}{4}.$$

In the previous example we had to write 3 as 3.00 and then perform the division. We chose to add two zeros to the integer 3 because we knew what the result was before performing the division. In general we need to add enough zeros to complete the division. We know the division process is complete when one of two things occurs.

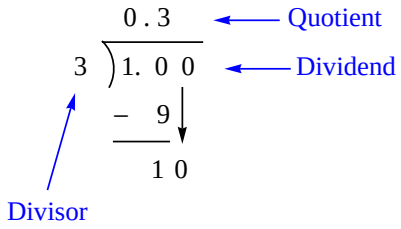
If the remainder is zero and there are no non-zero digits left in the dividend to "bring down" the division is complete. If the division process produces a never ending repeating quotient the division process can be stopped.

The result of the second option is a repeating decimal. This is not a problem. In chapter 3 we defined the set of Rational Numbers to be the set of numbers that can be written as a fraction. An alternative definition of a Rational Number is any number whose decimal representation is either terminating or repeating. A fraction with a repeating decimal representation is shown in the next example. The result of example 1 is a terminating decimal.

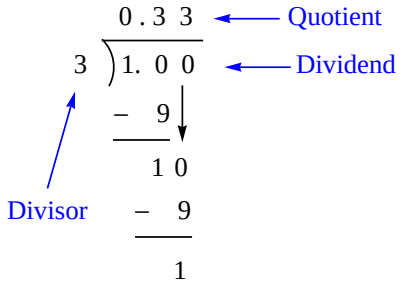
EXAMPLE 2 Write the fraction $\frac{1}{3}$ as a decimal.

SOLUTION: Here we need to divide 1 by 3.

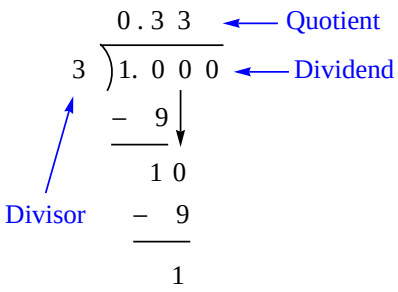
Because 3 does not "go into" 1, we need to rewrite the integer 1 as a decimal. In other words, write 1 as 1.00 and then perform the division.



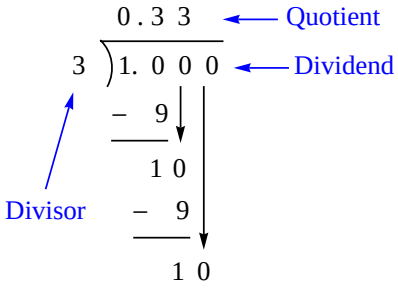
1. The divisor is 3 and the dividend is 1.00.
2. Set up the division as usual. Keep the decimal point of the quotient lined up with the decimal point of the dividend.
3. 3 does not "go into" 1 but it does "go into" 10, 3 times.
4. Subtract $3 \cdot 3$ from 10: $10 - 3 \cdot 3 = 1$.
5. Bring down the next digit in the dividend.



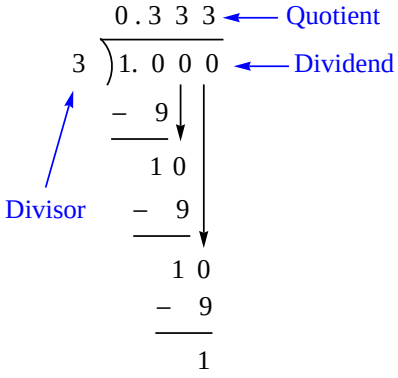
6. Continuing the division process, 3 goes into 10, 3 times.
7. Subtract $3 \cdot 3$ from 10: $10 - 3 \cdot 3 = 1$.



8. At this point the remainder is not zero and there is not another digit to bring down from the dividend. This is not an issue because we can simply add on another zero to the dividend. 1.00 and 1.000 are the same number.



9. Bring down the next digit in the dividend and continue the division process.



10. 3 goes into 10, 3 times.

11. Subtract $3 \cdot 3$ from 10: $10 - 3 \cdot 3 = 1$.

At this point we notice that at every step in the division we are calculating $10 - 3 \cdot 3 = 1$ and that this will continue on indefinitely. This means that the quotient will be $0.3333\ldots$.

To indicate that the 3 will repeat forever in the decimal we write $\frac{1}{3} = 0.\overline{3}$

In the previous example, the notation $0.\overline{3}$ was used to indicate that the digit 3 repeats indefinitely in the decimal representation.

In general, the notation indicates that the digits below the line will repeat indefinitely. That is,

$$2.\overline{125} = 2.125125125125125\ldots, \quad \text{and} \quad 0.361\overline{87} = 0.3618787878787\ldots$$

Certain repeating decimal representations are more common than others and it is sometimes useful to recognize, or utilize, a common form when you can. As an example, we can use the result of the previous example to write the decimal representation of $\frac{2}{3}$. That is,

$$\frac{2}{3} = 2\left(\frac{1}{3}\right) = 2(0.\overline{3}) = 0.\overline{6}.$$

Quick Note

Repeating decimals displayed on a calculator are rounded. This is because the display cannot show an infinite number of digits. For example, the number $2.\overline{6}$ may appear as 2.6666667.

Another common decimal representation of a fraction is

$$\frac{1}{9} = 0.\overline{1}.$$

We can use this result to write the decimal representation of $\frac{2}{9}$ and $\frac{4}{9}$. That is,

$$\frac{2}{9} = 2\left(\frac{1}{9}\right) = 2(0.\overline{1}) = 0.\overline{2}$$

and

$$\frac{4}{9} = 4\left(\frac{1}{9}\right) = 4(\overline{1}) = 0.\overline{4}$$

This can be continued to find the decimal representations for $\frac{5}{9}$, $\frac{7}{9}$ and $\frac{8}{9}$.

Section 5.3 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 10, write the given decimal as a fraction.

1) 0.25 2) 1.62 3) 2.75 4) 0.8 5) 4.55

6) 9.38 7) 0.02 8) 0.194 9) 5.46 10) 0.5

For problems 11 - 15, write the given fraction as a decimal.

11) $\frac{1}{2}$ 12) $\frac{2}{5}$ 13) $\frac{1}{6}$ 14) $\frac{3}{8}$ 15) $\frac{2}{9}$

For problems 16 - 18, write the given fraction as a decimal rounded to the indicated place value.

16) $\frac{1}{7}$; thousandths 17) $\frac{1}{11}$; hundredths 18) $\frac{3}{13}$; hundred thousandths

Review. Perform the indicated operation(s).

19) Round 13.67042 to the nearest tenths.

20) Round 1.0876 to the nearest thousandths.

21) Round 0.083 to the nearest hundredths.

22) $-\frac{1}{3}+\frac{5}{9}$ 23) $\frac{2}{7}-\frac{4}{3}$ 24) $-\frac{5}{2}-\frac{7}{8}$

25) $8.7602-12.34$ 26) $6.74+0.8073$ 27) $-3.445+2.6-1.02$

28) $\frac{3}{5}\left(\frac{7}{6}\right)$ 29) $-\frac{8}{9}\div\frac{7}{15}$ 30) $\frac{1}{3}\div\frac{2}{5}\div\frac{3}{7}$

31) Find the perimeter of a rectangle with sides of lengths $5\frac{1}{8}$ feet and $8\frac{3}{4}$ feet.

32) Max just took a test in Physics. The result of the test showed that $\frac{1}{5}$ of the class earned a C, $\frac{1}{7}$ of the class earned a B, and $\frac{1}{10}$ of the class earned an A. What fraction of the class passed the test.

33) What has to be true in order to add or subtract fractions?

34) How is division by a fraction accomplished?

Chapter 5
Section 5.3 – Converting Decimals to Fractions and Fractions to Decimals

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

1) _____

2) _____

3) _____

4) _____

5) _____

6) _____

7) _____

8) _____

9) _____

10) _____

11) _____

12) _____

13) _____

14) _____

15) _____

16) _____

17) _____

18) _____

19) _____

20) _____

21) _____

22) _____

23) _____

24) _____

25) _____

26) _____

27) _____

28) _____

29) _____

30) _____

31) _____

32) _____

33) _____

34) _____

5.4 Multiplication and Division with Decimals

5.4.1 Multiplication with Decimals

Suppose we wish to calculate the product $(1.4)(0.03)$. To put this another way, we wish to calculate

$$(14 \text{ tenths})(3 \text{ hundredths}).$$

If we write the decimals as fractions, without simplifying, we have

$$(14 \text{ tenths})(3 \text{ hundredths}) = \left(\frac{14}{10}\right)\left(\frac{3}{100}\right)$$

$$= \frac{14 \cdot 3}{10 \cdot 100}$$

$$= \frac{42}{1,000}.$$

The result of multiplying "14 tenths" and "3 hundredths" is "42 thousandths." Thus,

$$(1.4)(0.03) = 0.042.$$

Note the number of decimal places in the product is the sum of the number of decimal places in each factor. The number 1.4 contains one decimal place and the number 0.03 contains two. The product, 0.042, contains three decimal places $(1+2=3)$.

As another example, calculate $0.0052(0.16)$. Since 0.0052 is "52 ten-thousandths" and 0.16 is "16 hundredths" we can write this as

$$0.0052(0.16) = \frac{52}{10,000}\left(\frac{16}{100}\right).$$

Again, if we do not simplify the fractions and calculate the product we get the following.

$$\frac{52}{10,000}\left(\frac{16}{100}\right) = \frac{52 \cdot 16}{10,000 \cdot 100}$$

$$= \frac{832}{1,000,000}$$

$$= 832 \text{ millionths}$$

"832 millionths" means that the decimal representation should have six decimal places. Therefore,

$$0.0052(0.16) = 0.000832.$$

Note again that the number of decimal places in the product equals the sum of the decimal places in the factors. That is, since 0.0052 contains 4 decimal places and 0.16 contains 2, the product should contain $4+2=6$ decimal places.

The steps to multiply decimals can be summed up as follows.

Multiplying Decimals

To find the product of two decimals,

- ignore the decimal points and multiply as if multiplying two integers,
- the number of decimal places (digits to the right of the decimal point) in the product will be the sum of the decimal places of the factors.

EXAMPLE 1 Perform the indicated operation(s): $5.44(3.1)$

SOLUTION: For this problem, "ignoring the decimal points and multiply as if multiplying integers" means we need to multiply 544 and 31. Then since 5.44 contains two decimal place values and 3.1 contains one, we need to make sure the product contains three decimal place values.

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ \ \ } \\ 5\ 4\ 4 \\ \times\quad 3\ 1 \\ \hline 5\ 4\ 4 \\ +1\ 6\ 3\ 2\ 0 \\ \hline 1\ 6\ 8\ 6\ 4 \end{array}$$

Therefore,

$$5.44(3.1) = 16.864.$$

EXAMPLE 2 Perform the indicated operation(s): $0.045(1.35)$

SOLUTION: $45 \cdot 135 = 6,075$ and since 0.045 has 3 decimal places and 1.35 has 2 decimal places the product should contain 5 digits to the right of the decimal point. Therefore,

$$0.045(1.35) = 0.06075.$$

Multiplying by Powers of 10

There are some applications that require multiplying numbers by powers of 10. Consider the following examples. We first look at the product of 3.14 and 100.

$$3.14(100)$$

$$= \left(\frac{314}{100}\right)(100)$$

$$= 314\left(\frac{100}{100}\right)$$

$$= 314$$

Note that multiplying 3.14 by $100 = 10^2$ moved the decimal point to the right 2 places.

Next, consider the product of 0.06892 and $1,000$.

$$0.06892(1,000) = \left(\frac{6,892}{100,000}\right)(1,000)$$

$$= 6,892\left(\frac{1,000}{100,000}\right)$$

$$= 6,892\left(\frac{1,000}{1,000 \cdot 100}\right)$$

$$= 6,892\left(\frac{1}{100}\right)$$

$$= 68.92$$

Note that multiplying 0.6892 by $1,000 = 10^3$ moved the decimal place to the right three units.

In general, multiplying any number by 10^n for any positive integer n moves the decimal point to the right n places. We will show next that multiplying any number by 10^m for negative integers m will move the decimal place to the left m places.

Let's multiply 52.104 by $10^{-2} = \frac{1}{100}$. Since 52.104 is really $52,104$ thousandths we can write

$$52.104 = \frac{52,104}{1,000}.$$

Then, multiplying this by $10^{-2} = \frac{1}{100}$ gives

$$\frac{52,104}{1,000}\left(\frac{1}{100}\right)$$

$$= \frac{52,104}{1,000(100)}$$

$$= \frac{52,104}{100,000}.$$

This of course means $52,104$ hundred thousandths which is

$$0.52104$$

and we see that multiplying by 10^{-2} moved the decimal place to the left two units. This can be generalized as follows.

Multiplying by Powers of 10
Given any Real Number a,
• multiplying a by 10^n for any positive integer n moves the decimal point to the right n places, • multiplying a by 10^m for any negative integer m moves the decimal point to the left m places.

EXAMPLE 3 Perform the indicated operation(s): $12.004(10^4)$.

SOLUTION: Because we are multiplying by 10^4, we need to move the decimal point to the right 4 units. Therefore,

$$12.004(10^4) = 120,040.$$

EXAMPLE 4 Perform the indicated operation(s): $4,321.06(10^-5)$.

SOLUTION: Because we are multiplying by 10^{-5}, we need to move the decimal point to the left 5 units. Therefore,

$$4,321.06(10^{-5}) = 0.0432106.$$

5.4.2 Division of Decimals

Suppose now that we wish to divide 1.4 by 0.03. To perform the calculation we will use what we know about division by integers. To see how this works note the following.

The divisor in this example is 0.03. Writing this as a fraction we get

$$0.03 = \frac{3}{100}.$$

This means that if we multiply 0.03 by 100 we will get an integer. That is,

$$0.03(100) = \frac{3}{100}\left(\frac{100}{1}\right) = 3.$$

Now, remembering that we can multiply any number by 1 without changing it, and that any number divided by itself is 1, we can do the following manipulations.

$$\frac{1.4}{0.03} = \left(\frac{1.4}{0.03}\right)\left(\frac{100}{100}\right) = \frac{1.4(100)}{0.03(100)} = \frac{140}{3}.$$

We can now perform the division using the methods of section 5.3. That is,

$$\frac{1.4}{0.03} = \frac{140}{3} = 46.\overline{6}.$$

Now consider calculating $0.0405 \div 1.5$. The divisor can be written as

$$1.5 = \frac{1.5}{10} \quad \text{which then implies that} \quad 1.5(10) = \frac{15}{10}\left(\frac{10}{1}\right) = 15.$$

Performing the same manipulations as above we get

$$0.0405 \div 1.5 = \frac{0.0405}{1.5}$$

$$= \left(\frac{0.0405}{1.5}\right)\left(\frac{10}{10}\right)$$

$$= \frac{0.0405 \cdot 10}{1.5 \cdot 10}$$

$$= \frac{0.405}{15}.$$

Again using the techniques from section 5.3 we see that

$$0.0405 \div 1.5 = 0.405 \div 15 = 0.027.$$

In both examples we multiplied the dividend and the divisor by the power of 10 that made the divisor an integer and then performed the division.

Dividing Decimals
To find the quotient of two decimals,
• multiply the dividend and divisor by the power of 10 that makes the divisor an integer, • perform the division.

EXAMPLE 1 Perform the indicated operation(s): $0.275 \div 0.25$

SOLUTION: The divisor is 0.25. As a fraction, the divisor can be written as

$$\frac{25}{100}.$$

This means that we would need to multiply it by $100 = 10^2$ to make it an integer. So,

$$\frac{0.275}{0.25} = \left(\frac{0.275}{0.25}\right)\left(\frac{100}{100}\right)$$

$$= \frac{27.5}{25}.$$

Dividing 27.5 by 25 gives 1.1 so

$$0.275 \div 0.25 = 1.1.$$

EXAMPLE 2 Perform the indicated operation(s): $1.549 \div -0.002$

SOLUTION: First recall that a positive number divided by a negative number results in a negative number. We will keep this in mind for the final product but will ignore the signs during the calculations.

The divisor is 0.002. As a fraction it can be written as

$$\frac{2}{1,000}.$$

This means that we would need to multiply it by $1,000 = 10^3$ to make it an integer.

So,

$$\frac{1.549}{0.002}$$

$$= \left(\frac{1.549}{0.002}\right)\left(\frac{1,000}{1,000}\right)$$

$$= \frac{1,549}{2}.$$

Dividing $1,549$ by 2 gives 774.5 so

$$1.549 \div -0.002 = -774.5.$$

5.4.3 Applications: Unit Conversion

A straightforward application of decimal multiplication and division is converting between units. Here we take advantage again of the idea that multiplying any number by one does not change the number. One of the new things we take a look at here is in keeping track of units.

EXAMPLE 1 Convert 5.2 feet into inches.

SOLUTION: For this problem we need to know that 12 inches equals 1 foot. This means that

$$\frac{12 \text{ in}}{1 \text{ ft}} = 1.$$

Again, because we can multiply any number by one without changing it we can do the following.

$$5.2 \text{ ft} = (5.2 \text{ ft}) \left(\frac{12 \text{ in}}{1 \text{ ft}} \right).$$

Avoiding Mistakes

Units of measurement must be included for the answer to be correct.

Keeping track of the units and remembering that anything (other than zero) divided by itself is one we see that

$$5.2 \text{ ft} = (5.2 \text{ ft}) \left(\frac{12 \text{ in}}{1 \text{ ft}} \right).$$

$$= \frac{(5.2 \text{ ft})(12 \text{ in})}{1 \text{ ft}}$$

$$= \frac{(5.2 \cancel{\text{ft}})(12 \text{ in})}{1 \cancel{\text{ft}}}$$

$$= 5.2(12 \text{ in})$$

$$= 62.4 \text{ in}.$$

We know we did the correct conversion because in the above solution the units “ft” cancelled and we were left with the units we wanted.

EXAMPLE 2 Convert 148 inches into feet.

SOLUTION: For this problem we again want to use that 12 inches equals 1 foot but we want to use

$$\frac{1 \text{ ft}}{12 \text{ in}} = 1.$$

The reason we want this "conversion factor" is that we now want the units "inches" to cancel and leave us with the units "feet."

$$148 \text{ in} = 148 \text{ in}\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)$$

$$= 148 \cancel{\text{in}}\left(\frac{1 \text{ ft}}{12 \cancel{\text{in}}}\right)$$

$$= \frac{148 \text{ ft}}{12}$$

When we perform the division we get $12.\overline{3}$ ft. Remembering that $0.\overline{3} = \frac{1}{3}$ we could write this as $12\frac{1}{3}$ ft it it were helpful.

Other important conversions are between metric units. The following table of metric prefixes will be helpful in converting between metric units.

Metric Prefixes		
Kilo: 10^3	$10^0 = 1$	Milli: 10^{-3}
Hecto: 10^2	Deci: 10^{-1}	Micro: 10^{-6}
Deca: 10^1	Centi 10^{-2}	Nano: 10^{-9}

The next two examples illustrate how to use the prefix table to convert between metric units.

EXAMPLE 3 Convert 25 centimeters to meters.

SOLUTION: From the prefix table we see that the prefix "centi" implies 10^{-2}. This means that

$$1 \text{ centimeter} = 10^{-2} \text{ meter} \Rightarrow 1 \text{ cm} = 10^{-2} \text{ m}.$$

For this conversion we wish to convert centimeters to meters. This means in the solution process we want the units "centimeters' " to cancel. Therefore the conversion factor we should use for this problem is

$$1 = \frac{10^{-2} \text{ m}}{1 \text{ cm}}.$$

Converting we get

$$25 \text{ cm} = 25 \text{ cm} \left(\frac{10^{-2} \text{ m}}{1 \text{ cm}} \right)$$

$$= 25 \cancel{\text{cm}} \left(\frac{10^{-2} \text{ m}}{1 \cancel{\text{cm}}} \right)$$

$$= 25(10^{-2} \text{ m})$$

$$= 0.25 \text{ m}.$$

EXAMPLE 4 Convert 43 liters to kiloliters.

SOLUTION: From the prefix table we see that the prefix "kilo" implies 10^3. This means that

$$1 \text{ kiloliter} = 10^3 \text{ liters} \Rightarrow 1 \text{ kL} = 10^3 \text{ L}.$$

Because we are being asked to convert liters to kiloliters we want the units "liters" to cancel. This means that the conversion factor should be

$$1 = \frac{1 \text{ kL}}{10^3 \text{ L}}.$$

The conversion is then

$$43\ \text{L} = 43\ \text{L}\left(\frac{1\ \text{kL}}{10^3\ \text{L}}\right)$$

$$= 43\cancel{\text{L}}\left(\frac{1\ \text{kL}}{10^3\cancel{\text{L}}}\right)$$

$$= 43\left(\frac{1\ \text{kL}}{10^3}\right)$$

$$= 43(10^{-3}\ \text{kL})$$

$$= 0.043\ \text{kL}.$$

While there are many other conversions we could practice with there are too many to list here. During the problem set when it is not clear what the conversion factor should be it will be provided.

Section 5.4 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 4, determine the number of decimal places in the product without performing the multiplication.

1) $0.34(4)$ 2) $4.1(9.13)$ 3) $0.1004(5.318)$ 4) $12.591(6.38)$

5) $-14.5(0.468)$

For problems 6 - 9, perform the multiplication.

6) $3.14(5)$ 7) $12(0.136)$ 8) $0.2(10.4)$ 9) $5.9(4.1)$

For problems 10 - 13, perform the multiplication by moving the decimal point.

10) $11.34(100)$ 11) $1,000(12.1)$ 12) $41.56(10^{-2})$ 13) $8.072(10^{-3})$

For problems 14 - 17, perform the indicated division.

14) $0.05 \div 0.002$ 15) $1.09 \div 0.05$ 16) $3 \div 1.2$ 17) $4.026 \div 0.04$

For problems 18 - 22, perform the indicated conversion.

18) Convert 258 centimeters (cm) to meters (m). Use the table of prefixes provided in the section to determine the conversion factor.

19) Convert 17 yards (yd) to feet (ft). Use the fact that 3 feet equals 1 yard to determine the conversion factor.

20) Convert 3.09 decaliters (daL) to liters (L). Use the table of prefixes provided in the section to determine the conversion factor.

21) Convert 2.5 pounds (lbs) to ounces (oz). Use the fact that 16 ounces equals 1 pound to determine the conversion factor.

22) Convert 5.379 meters (m) to nanometers (nm). Use the table of prefixes provided in the section to determine the conversion factor.

For problems 23 - 25, perform the indicated operation(s). Write your answer as a decimal.

23) $3.61 + 9.101 - 1.42$

24) $10.552 - 9.138 + 0.256$

25) $\frac{3}{8} + \frac{9}{8}$

For problems 26 - 31, perform the indicated operation(s). Write your answer as a rational number.

26) $\frac{10}{27} - \frac{1}{27}$

27) $\frac{3}{7} + \frac{5}{14}$

28) $1\frac{5}{12} - \frac{7}{9}$

29) $\frac{21}{20} \cdot \frac{38}{15}$

30) $\frac{27}{100} \cdot \frac{25}{63}$

31) $\frac{3}{4} \div 5\frac{1}{3}$

For problems 32 - 34, convert the mixed numbers to improper fractions and then perform the indicated operation.

32) $\left(5\frac{1}{4}\right)\left(6\frac{2}{9}\right)$

33) $\left(12\frac{1}{2}\right) \div \left(2\frac{1}{7}\right)$

34) $\left(1\frac{2}{3}\right) \div \left(2\frac{1}{4}\right)$

Chapter 5
Section 5.4 – Multiplication and Division with Decimals

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

1) _____

2) _____

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10) _____

11) _____

12) _____

13) _____

14) _____

15) _____

16) _____

17) _____

18) _____

19) _____

20) _____

21) _____

22) _____

23) _____

24) _____

25) _____

26) _____

27) _____

28) _____

29) _____

30) _____

31) _____

32) _____

33) _____

34) _____

PERCENTS AND THEIR APPLICATIONS

CHAPTER

6

CHAPTER OBJECTIVES

6.1 Percents as Fractions and Decimals

6.2 Fractions and Decimals as Percents

6.3 Percent Proportions and Applications

6.4 More Applications of Percent Proportions

Holding The Lead

For some observers the question is, “Is this ever going to end?” But most sports fans want to know, “How is this going to end?” A team of researchers used a mathematical idea known as a random walk to model the way scoring leads grow and shrink over the course of a game in a competitive team sport. They found that the likelihood that a lead of L points is “safe” (that is, will not be eliminated) with t seconds left in the game and can be calculated with expressions involving exponential and trigonometric functions. This formula predicts that in basketball, a lead of 10 points with about eight minutes remaining, or of 18 points at halftime, is 90% safe.

It may seem strange that important game characteristics could be associated with anything random, especially given the skill levels of professional players. Yet, using comprehensive data on more than 10,000 professional basketball games, the researchers showed that their formula is remarkably accurate, correct about 95% of the time. The researchers also tested the formula with data from several other sports and showed that its accuracy declines slightly as the number of points and/or goals scored in a game decreases.

6.1 Percents as Fractions and Decimals

The word "Percent" means "per one hundred." That is, when we say "twenty five percent" we mean "25 per one hundred" which is then translated to mean

$$\frac{25}{100}.$$

The notation we use to denote percent is the symbol %. This means that 45% is interpreted to mean

45 per one hundred or 45 out of one hundred.

6.1.1 Converting Percents to Fractions

Because the term percent means per one hundred, we can convert a percent to a fraction simply based on definition.

EXAMPLE 1 Convert 56% to a fraction.

SOLUTION: By definition,

$$56\% = \frac{56}{100}.$$

Simplifying gives

$$\frac{56}{100} = \frac{4(14)}{4(25)}$$

$$= \frac{\cancel{4}(14)}{\cancel{4}(25)}$$

$$= \frac{14}{25}.$$

Therefore, $56\% = \frac{14}{25}$.

EXAMPLE 2 Convert 35% to a fraction.

SOLUTION: By definition,

$$35\% = \frac{35}{100}.$$

Simplifying gives

$$\frac{35}{100} = \frac{5(7)}{5(20)}$$

$$= \frac{\cancel{5}(7)}{\cancel{5}(20)}$$

$$= \frac{7}{20}.$$

Therefore, $35\% = \frac{7}{20}$.

EXAMPLE 3 Convert $33\frac{1}{3}\%$ to a fraction.

SOLUTION: We can convert the mixed number to an improper fraction and then use the definition of percent.

$$33\frac{1}{3} = \frac{33(3)+1}{3} = \frac{100}{3}$$

By definition,

$$33\frac{1}{3}\% = \frac{100}{3}\%$$

$$= \frac{\frac{100}{3}}{100}$$

$$= \frac{100}{3}\left(\frac{1}{100}\right)$$

$$= \frac{100}{3(100)}$$

$$= \frac{1}{3}.$$

Therefore, $33\frac{1}{3}\% = \frac{1}{3}$.

6.1.2 Converting Percents to Decimals

Again, because the term percent means per one hundred, we can convert a percent to a decimal based solely on the definition.

EXAMPLE 1 Convert 32% to a decimal.

SOLUTION: By definition, 32% means "32 per one hundred." That is,

$$32\% = \frac{32}{100}.$$

We know that $\frac{32}{100}$ is "32 hundredths." Therefore,

$$32\% = \frac{32}{100} = 0.32.$$

EXAMPLE 2 Convert 87.7% to a decimal.

SOLUTION: Again, by definition

$$87.7\% = \frac{877}{1000}$$

which means "877 thousandths." Therefore,

$$87.7\% = \frac{877}{1000} = 0.877.$$

EXAMPLE 3 Convert 0.06% to a decimal.

SOLUTION: If we ignore the % for the moment, recall that the decimal

$$0.06 = \frac{6}{100}.$$

Then, since percent means per one hundred,

$$0.06\% = \frac{6}{100}\%$$

$$= \frac{\frac{6}{100}}{100}$$

$$= \frac{6}{100}\left(\frac{1}{100}\right)$$

$$= \frac{6}{100(100)}$$

$$= \frac{6}{10,000}$$

Therefore, $0.06\% = \frac{6}{10,000}$ = "6 ten thousandths" and so,

$$0.06\% = 0.0006.$$

Note, to convert a percent to a decimal we replace the % symbol with division by 100 which is equivalent to multiplying by 10^{-2}. From the previous section we know this moves the decimal point to the left two place values. From the previous examples,

$$32\% = 32.0\% = 0.32,\ 87.7\% = 0.877,\ \text{ and }\ 0.06\% = 0.0006.$$

Helpful Hint: Converting Percents to Decimals

To convert a percent to a decimal, drop the % symbol and move the decimal place to the left two place values.

EXAMPLE 5 Convert $2\frac{3}{4}\%$ to a decimal.

SOLUTION: First convert the mixed number to a decimal.

$$2\frac{3}{4} = \frac{2(4)+3}{4} = \frac{11}{4}$$

and

$$\frac{11}{4} = 11 \div 4 = 2.75.$$

Now, $2\frac{3}{4}\% = 2.75\%$. To convert to a decimal, move the decimal place to the left two place values. That is,

$$2\frac{3}{4}\% = 2.75\% = 0.0275.$$

6.1.3 Applications

In order to use percents in applications there are two things to keep in mind. The first is that we will always convert the percent to a decimal in any calculations that need to be performed. The second thing we need to remember is what the phrase "$A\%$ of B" means.

In application, the phrase "$A\%$ of B" means "multiply $0.0A$ and B." In other words, it means convert the percent to a decimal and multiply it by B.

As an example, the phrase "25% of 44" implies the multiplication $0.25(44)$ and since

$$0.25(44) = 11,$$

we say that "25% of 44 is 11."

EXAMPLE 1 Out of a truckload of apples, 15% of the apples are usually bad. If a truck contains 200 apples, how many apples are bad?

SOLUTION: To answer this question we need to translate the phrase "15% of the apples." Since there are 200 apples in the truck, we know that

$$15\% \text{ of the apples} = 15\% \text{ of } 200.$$

Because "15% of 200" means multiply 15% and 200, and

$$15\%(200) = 0.15(200) = 30,$$

30 out of the 200 apples will be bad.

Avoiding Mistakes

It is always a good idea to check your answer in the context of the problem to determine if your answer is reasonable.

Some applications with percents can be solved by just using the definition of percent.

EXAMPLE 2 Nearly 66 of every 100 acres of Amazon Rainforest is found in Brazil. What percent it this?

SOLUTION: Since we are given that there is 66 acres out of every 100, and this can be rewritten as "66 per 100 acres", we know that

$$\frac{66}{100} = 66\%.$$

Section 6.1 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 11, convert the given percent to a simplified fraction.

1) 7%
2) 55%
3) 150%
4) 73%
5) 8%
6) 5.4%
7) 115%
8) 36.5%
9) 340%
10) 99.9%
11) $2\frac{1}{2}\%$

For problems 12 - 21, convert the given percent to a decimal.

12) 47%
13) 76%
14) 23.8%
15) 170%
16) 5.95%
17) 200.3%
18) 4.028%
19) $7\frac{3}{4}\%$
20) $39\frac{3}{5}\%$
21) $\frac{1}{10}\%$

22) Define the term "percent."

23) In a class of 100 students, there are 45 male students. What percentage of the students are male students?

24) If 47 out of every 100 drivers regularly use seatbelts, what percentage of drivers regularly use seatbelts?

25) 72% of the residents of Riverside Heights are older than 60 years. If the population of Riverside Heights is 40,000 people, how many of them are older than 60?

26) 30% of the players in a softball league are left-handed. There are 120 players in this league. How many players are left-handed?

27) Match each percent with its fraction form. 4%, 25%, 250%, 80%.

a) $\frac{1}{4}$ b) $\frac{4}{5}$ c) $\frac{1}{25}$ d) $\frac{5}{2}$

28) Match each percent with its decimal form. 60%, $66\frac{2}{3}$%, 6%.

a) 0.06 b) 0.6 c) $0.\overline{6}$

Find the equivalent decimal and fraction forms of the percents given in problems 28 and 29.

29) The unemployment rate for Habershaw County is 13%.

30) During the year 2015, the stock's value increased by 20.5%.

Chapter 6
Section 6.1 – Percents as Fractions and Decimals

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

1) _____

2) _____

3) _____

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6) _____

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11) _____

12) _____

13) _____

14) _____

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16) _____

17) _____

18) _____

19) _____

20) _____

21) _____

22) _____

23) _____

24) _____

25) _____

26) _____

27) a) _____

b) _____

c) _____

d) _____

28) a) _____

b) _____

c) _____

29) _____

30) _____

6.2 Fractions and Decimals as Percents

In the previous section you learned how to convert percents to fractions and decimals. In this section you will learn how to convert fractions and decimals to percents.

Similar to the previous section, the conversion from a fraction or decimal to a percent relies on the definition of a percent.

Consider the problem of converting 72% to a decimal. Because "percent" means "per one hundred,"

$$72\% = \frac{72}{100} = 0.72.$$

So to convert a percent to a decimal or fraction, we replaced the % symbol with division by 100.

To go the other direction, to convert a fraction or decimal to a percent, we will multiply by 100%.

6.2.1 Convert Decimals to Percents

EXAMPLE 1 Convert 0.08 to a percent.

SOLUTION: First observe that

$$0.08 = \frac{8}{100}.$$

Then, by the definition of percent,

$$0.08 = \frac{8}{100} = 8\%.$$

As mentioned earlier, we could just multiply by 100% to get

$$0.08 \Rightarrow 0.08(100\%) = \left(\frac{8}{100}\right)(100)(\%) = 8\%.$$

EXAMPLE 2 Convert 0.67 to a percent.

SOLUTION: Observe again that

$$0.67 = \frac{67}{100} = 67\%.$$

Alternatively,

$$0.67 \Rightarrow 0.67(100\%) = \left(\frac{67}{100}\right)(100)(\%) = 67\%.$$

Notice from the previous examples that to convert a decimal to a percent, multiplying by 100 moves the decimal point to the right two place values.

Helpful Hint: Converting Decimals to Percents

To convert a decimal to a percent, move the decimal place to the right two place values and add the % symbol.

EXAMPLE 3 Convert 2.43 to a percent.

SOLUTION: Observe again that

$$2.43 = \frac{243}{100} = 243\%.$$

Alternatively,

$$2.43 \Rightarrow 2.43(100\%)$$

$$= \left(\frac{243}{100}\right)(100)(\%)$$

$$= 243\%.$$

6.2.2 Convert Fractions to Percents

To convert a fraction to a percent we will again multiply by 100%.

EXAMPLE 1 Convert $\frac{1}{4}$ to a percent.

SOLUTION: To convert a fraction to a percent, multiply by 100%.

$$\frac{1}{4} \Rightarrow \left(\frac{1}{4}\right)(100\%)$$

$$= \frac{100}{4}\% = 25\%.$$

From this example we note that

$$\frac{1}{4},\ 0.25,\ \text{and}\ 25\%$$

all represent the same value.

EXAMPLE 2 Convert $\frac{7}{9}$ to a percent.

SOLUTION: To convert a fraction to a percent, multiply by 100%.

$$\frac{7}{9} \Rightarrow \left(\frac{7}{9}\right)(100\%)$$

$$= \frac{700}{9}\%.$$

Dividing 700 by 9 gives

$$\frac{700}{9}\% = 77.\overline{7}\%.$$

Notice that we could have written this as the mixed number $77\frac{7}{9}\%$.

Note that in these examples we could have first converted the given fraction to a decimal and then converted the decimal to a percent. The next example illustrates this technique.

EXAMPLE 3 Convert $1\frac{1}{6}$ to a percent.

SOLUTION: Convert the given mixed number into a decimal.

$$1\frac{1}{6} = \frac{7}{6} = 1.1\overline{6}.$$

To convert this to a percent multiply by 100%. Remember, multiplying by 100 moves the decimal point to the right two place values.

$$1\frac{1}{6} = 1.16\overline{6} \Rightarrow 1.16\overline{6}(100\%) = 116.\overline{6}\%.$$

6.2.3 Converting and Rounding Percents

EXAMPLE 1 Write $\frac{5}{13}$ as a percent rounded to the nearest tenth of a percent.

SOLUTION: In order to round to a desired percent place value we need to first write the given fraction or decimal as a percent. For this example we can write the fraction as a decimal, convert the decimal to a percent and then round the percent to the indicated place value.

Dividing 5 by 13 gives

$$\frac{5}{13} = 0.\overline{384615}.$$

As a percent,

$$\frac{5}{13} = 38.4615\overline{384615}\%.$$

We can now round the percent to the nearest tenth of a percent. Therefore,

$$\frac{5}{13} \approx 38.5\%.$$

EXAMPLE 2 Write $\frac{7}{12}$ as a percent rounded to the nearest percent.

SOLUTION: To round to an indicated percent place value we need to first write the given number as a percent. For this problem we have

$$\frac{7}{12} = 0.58\overline{3} \Rightarrow 58.\overline{3}\%.$$

Rounded to the nearest percent we get

$$\frac{7}{12} \approx 58\%.$$

EXAMPLE 3 Write 3.567763 as a percent rounded to the nearest hundredth of a percent.

SOLUTION: Convert the decimal to a percent and then round the result the indicated percent place value.

$$3.567763 = 356.7763\%.$$

Rounded to the nearest hundredth of a percent gives 356.78%.

Section 6.2 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 8, convert the given decimal to a percent.

1) 3.4
2) 0.72
3) 0.008
4) 0.92
5) 0.77
6) 3.5
7) 0.0098
8) 1.773

For problems 9 - 17, convert the given fraction or mixed number to a percent.

9) $\frac{11}{5}$
10) $\frac{79}{100}$
11) $\frac{3}{4}$
12) $\frac{83}{100}$
13) $\frac{1}{8}$
14) $\frac{11}{10}$
15) $\frac{7}{9}$
16) $\frac{17}{20}$
17) $2\frac{4}{5}$
18) $5\frac{3}{10}$
19) $1\frac{1}{3}$
20) $3\frac{1}{9}$

The following phrases mention fractions or decimals. Convert the stated fraction or decimal to a percent.

21) One eighth of the rooms were vacant.
22) Three quarters of the senators voted.
23) Four fifths of the class received scholarships.
24) A third of the entree choices were seafood.
25) The probability is 0.125.
26) A magnification factor of 1.8 is needed.

For 27 - 30, write the fraction as a percent rounded to the nearest tenth of a percent.

27) $\frac{5}{6}$
28) $\frac{2}{7}$
29) $\frac{16}{17}$
30) $\frac{4}{3}$

31) Match each fraction with its equivalent percent. $\frac{17}{100}, \frac{17}{99}, \frac{17}{10}$

a) 170% b) 17% c) $17.\overline{17}\%$

32) Match each decimal with its equivalent percent. 4.3, 0.43, 0.043

a) 4.3% b) 43% c) 430%

Chapter 6
Section 6.2 – Fractions and Decimals as Percents

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

1) _____

2) _____

3) _____

4) _____

5) _____

6) _____

7) _____

8) _____

9) _____

10) _____

11) _____

12) _____

13) _____

14) _____

15) _____

16) _____

17) _____

18) _____

19) _____

20) _____

21) _____

22) _____

23) _____

24) _____

25) _____

26) _____

27) _____

28) _____

29) _____

30) _____

31) a) _____

b) _____

c) _____

32) a) _____

b) _____

c) _____

6.3 Percent Proportions and Applications

Recall that a percent is a fraction in terms of parts per 100. For example, 33% means "33 per 100 or as a fraction, $\frac{33}{100}$. Moreover, remember that equivalent fractions are fractions that represent the same quantity. That is,

$$50\% = \frac{50}{100} = \frac{1}{2}$$

so $\frac{50}{100}$ and $\frac{1}{2}$ are equivalent.

Setting two fractions equal to each other forms what is called a **proportion**. In this section we will be working with **percent proportions**. A percent proportion is formed by setting a percent equal to an equivalent fraction. As a simple example,

$$50\% = \frac{1}{2} \Rightarrow \frac{50}{100} = \frac{1}{2}$$

is a percent proportion.

6.3.1 Translating Percent Proportions

We will be using percent proportions to help solve applications so it will be crucial to understand the components of a percent proportion.

Percent Proportions

In general, a percent proportion is set up as

$$\frac{Amount}{Base} = Percent.$$

In practice, the *Percent* (sometimes called the p value) is converted either to a fraction or a decimal. The *Base* is the total, or whole, amount given in the problem and the *Amount* is the part of the total being considered.

EXAMPLE 1 Set up an appropriate percent proportion for the following statement. "25% of 60 students is 15."

SOLUTION: For this problem the percent is 25% so we know we have

$$\frac{Amount}{Base} = 25\% \Rightarrow \frac{Amount}{Base} = \frac{25}{100} \Rightarrow \frac{Amount}{Base} = \frac{1}{4}.$$

We now need to determine the *Amount* and the *Base*. The definition of the *Base* is the total, or whole, amount given in the problem and the *Amount* is the part of the total being considered. For this problem, 60 is the total and 15 is the part of the total. Therefore,

$$\frac{Amount}{Base} = \frac{1}{4} \Rightarrow \frac{15}{60} = \frac{1}{4}.$$

EXAMPLE 2 Set up an appropriate percent proportion for the following statement. "$24 is 50% of $48."

SOLUTION: For this problem the percent is 50% so we know we have

$$\frac{Amount}{Base} = 50\% \Rightarrow \frac{Amount}{Base} = \frac{50}{100} \Rightarrow \frac{Amount}{Base} = \frac{1}{2}.$$

We now need to determine the *Amount* and the *Base*. The definition of the *Base* is the total, or whole, amount given in the problem and the *Amount* is the part of the total being considered. For this problem, $48 is the total and $24 is the part of the total. Therefore,

$$\frac{Amount}{Base} = \frac{1}{2} \Rightarrow \frac{24}{48} = \frac{1}{2}.$$

Helpful Hint: Translating Percent Proportions

Each of the previous examples contained statements similar to "*P*% of *the Base*."

In example 1 it was "25% of 60" and in example 2 it was"50% of $48."

This can be helpful in determining which quantity is the *Base* in a percent proportion.

EXAMPLE 3 Set up an appropriate percent proportion for the following statement. "75 is 125% of 60."

SOLUTION: Again we can begin the set up as

$$\frac{Amount}{Base} = 125\% \Rightarrow \frac{Amount}{Base} = \frac{125}{100} \Rightarrow \frac{Amount}{Base} = \frac{5}{4}.$$

In the statement it says "125% of 60." This means the *Base* must be 60 and so the *Amount* is 75.

$$\Rightarrow \frac{75}{60} = \frac{5}{4}.$$

6.3.2 Solving Percent Proportions

Now that we can set up percent proportions we begin looking at solving problems in which some part of the percent proportion is unknown. The process that we will use to find the missing parts of a percent proportion is that of rewriting fractions so that they have the same denominator, or numerator, depending on what we need.

EXAMPLE 1 What is 10% of 120 trees?

SOLUTION: Setting up a percent proportion to model this scenario we get

$$\frac{Amount}{Base} = Percent \Rightarrow \frac{Amount}{120} = 10\%.$$

Notice that the unknown quantity in the percent proportion is the *Amount*. To figure out what the *Amount* is, rewrite the percent proportion as

$$\frac{Amount}{120} = \frac{10}{100} \Rightarrow \frac{Amount}{120} = \frac{1}{10}$$

and rewrite the proposed equivalent fractions with the same denominator.

The LCD of 10 and 120 is 120. Therefore,

$$\frac{Amount}{120} = \frac{1}{10} \Rightarrow \frac{Amount}{120} = \left(\frac{1}{10}\right)\left(\frac{12}{12}\right) = \frac{12}{120}.$$

Because

$$\frac{Amount}{120} = \frac{12}{120},$$

it must be that *Amount* = 12. Therefore, 10% of 120 trees is 12 trees.

EXAMPLE 2 \$69 is 30% of how many dollars?

SOLUTION: Consider the part of the statement "30% of how many dollars?" This implies that the *Base* is unknown. Setting up the percent proportion gives

$$\frac{Amount}{Base} = Percent \Rightarrow \frac{69}{Base} = 30\%.$$

Converting the percent to a fraction we get

$$\frac{69}{Base} = \frac{30}{100} \Rightarrow \frac{69}{Base} = \frac{3}{10}.$$

Since these are equivalent fractions their numerators and denominators must be the same. Therefore,

$$\frac{69}{Base} = \left(\frac{3}{10}\right)\left(\frac{23}{23}\right) = \frac{69}{230}.$$

This implies that the *Base* = 230 and so, \$69 is 30% of \$230.

Helpful Hint: Solving Percent Proportions

The percent proportions in the previous examples contained an unknown part. In example 1 the unknown part was the *Amount* and in example 2 it was the *Base.*

As it can be cumbersome to keep writing a word or phrase for each step in a solution it is customary to use a letter to represent the unknown quantity.

EXAMPLE 3 What percent of 80 miles is 12.4 miles?

SOLUTION: The unknown quantity in this problem is the percent. Moreover, the phrase "What percent of 80" implies that the *Base* is 80. Therefore, the percent proportion is

$$\frac{12.4}{80} = \textit{Percent}.$$

Instead of writing *Percent* throughout the solution, let $p = \textit{Percent}$. Then,

$$\frac{12.4}{80} = \textit{Percent} \;\Rightarrow\; \frac{12.4}{80} = p\%.$$

This problem is different than the previous two since to find the unknown, we could just perform the division $12.4 \div 80$. Since

$$12.4 \div 80 = 0.155$$

the unknown percent must be 15.5%.

EXAMPLE 4 Yellowstone National Park has $3,366$ square miles of undeveloped land. If this area represents 99% of the total area of the park, find the total area of the park.

SOLUTION: We could restate this problem as "$3,366$ mi^2 is 99% of what total area?" Since the unknown quantity in the precent proportion is the *Base*, let $b = \textit{Base}$. The percent proportion can then be written as

$$\frac{3,366}{b} = 99\% \;\Rightarrow\; \frac{3,366}{b} = \frac{99}{100}.$$

Since these are equivalent fractions they must have the same numerators. Note that $3,366 \div 99 = 34$.

$$\frac{3,366}{b} = \frac{99}{100} \;\Rightarrow\; \frac{3,366}{b} = \left(\frac{9}{100}\right)\left(\frac{34}{34}\right) = \frac{3,366}{3400}.$$

Therefore, the total area of Yellowstone National Park is $3,400$ square miles.

Recall that a Rational Number is defined to be any number that can be written as a fraction $\frac{a}{b}$ where both a and b are integers and $b \neq 0$. In practice then we do not refer to something like

$$\frac{14.3}{90}$$

as a Rational Number or as a fraction. In practice, when referring to some general quantity over some other general quantity, use the term **ratio** rather than fraction.

This small but important distinction between fraction and ratio makes it possible for us to solve percent proportions like the next example.

EXAMPLE 5 56.7 is 45% of what number?

SOLUTION: Since the *Base* is the unknown, let $b = Base$. The percent proportion modeling this problem is

$$\frac{56.7}{b} = 45\% \;\Rightarrow\; \frac{56.7}{b} = \frac{9}{20}.$$

Since we want these ratios to be equivalent, and $56.7 \div 9 = 6.3$, we see that

$$\frac{56.7}{b} = \frac{9}{20}$$

$$\Rightarrow \frac{56.7}{b} = \left(\frac{9}{20}\right)\left(\frac{6.3}{6.3}\right)$$

$$\Rightarrow \frac{56.7}{b} = \frac{56.7}{126}.$$

and it must be that $b = 126$.

Section 6.3 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 5, identify the *Amount*, *Base*, and *Percent*.

1) 40% of \$500 is \$200.

2) 18 cars out of 25 cars is 72%.

3) 50 pounds is 5% of 1000 pounds.

4) 16 votes is 80% of 20 votes.

5) 6% tax on \$1000 purchase is \$60.

For problems 6 - 10, set up an appropriate percent proportion.

6) 90% of 400 dentists is 360 dentists.

7) 30 cats are 10% of 300 cats.

8) 20 ounces is 25% of 80 ounces.

9) 3% commission on \$150,000 sale is \$4500.

10) 48 yes votes out of 50 votes represent a 96% approval rate.

For problems 11 - 35, solve the percent proportions.

11) What number is 60% of 500?

12) What is 180% of 700?

13) What is $\frac{2}{5}$% of 30,000 service calls?

14) What is 46% of 300 applications.

15) A 20% tip on a \$68 bill would be how much?

16) 210 is 35% of what number?

17) 62.5 is 12.5% of what number?

18) The sales tax, \$270, represents 6% of what purchase price?

19) Frank watered 120 plants, which is 40% of the plants in his nursery. How many plants are there in his nursery?

20) 5.5% of the residents of Big Horn County are less than 10 years old. If 1100 people in Big Horn County are less than 10 years old, then what is the population of Big Horn County?

21) 280 is what percent of 800?

22) 12 is what percent of 10?

23) Marissa correctly answered 19 questions on a 25 question exam. What percent of the questions did she answer correctly?

24) $13,500$ tax returns were audited out of $450,000$ returns. What percent of tax returns were audited?

25) Justine made $120 in tips at a restaurant Friday. If her sales for the night totaled $480, what was the average percent tip she received?

26) What number is 72% of 48?

27) 28 is 16% of what number?

28) 3300 is what percent of $82,500$?

29) John makes a 4% commission on his sales at the clothing shop. How much would John make on commission if he sells $1800 for the week?

30) Pancho lost 35 pounds this year, which was 14% of his weight from previous year. How much did Pancho weigh the previous year?

31) Tina paid 16% of her taxable income ($58,700) for income taxes last year. How much did Tina pay for her income tax last year?

32) Jackson made 78 free throws out of 96 attempts taken over the season. Compute his free throw percentage.

33) Marcia bought a dress whose price tag read $120. The store had a clearance sale with a 60% discount on all merchandise. How much did she pay for the dress?

34) Brady completed 33 passes in a football game in which he completed 75% of his passes. How many passes did he attempt in the game?

35) Shanika earned $1200 in dividends from her stock for the year. If this stock paid a 5% dividend for the year, then how much did she have invested before receiving the dividends?

Chapter 6
Section 6.3 – Percent Proportion and Application

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

1) Amount: _____ Base: _____ Percent: _____

2) Amount: _____ Base: _____ Percent: _____

3) Amount: _____ Base: _____ Percent: _____

4) Amount: _____ Base: _____ Percent: _____

5) Amount: _____ Base: _____ Percent: _____

6) _____

7) _____

8) _____

9) _____

10) _____

11) _____

12) _____

13) _____

14) _____

15) _____

16) _____

17) _____

18) _____

19) _____

20) _____

21) _____

22) _____

23) _____

24) _____

25) _____

26) _____

27) _____

28) _____

29) _____

30) _____

31) _____

32) _____

33) _____

34) _____

35) _____

6.4 More Applications of Percent Proportions

In this section we will learn how to work with two common applications involving percents, sales tax and commission.

6.4.1 Sales Tax

The amount of sales tax is some percent of the cost of merchandise. As in the previous section we can handle sales tax applications with percent proportions. In the previous section we defined a percent proportion as

$$\frac{Amount}{Base} = Percent.$$

In this section we can apply this idea directly to sales tax by rewriting the percent proportion as

$$\frac{Amount\ of\ Sales\ Tax}{Cost\ of\ Merchandise} = Tax\ Rate$$

where the *Tax Rate* is a percentage.

We use sales tax to calculate the total amount of a purchase. That is,

$$\text{Total Amount of Purchase} = Cost\ of\ Merchandise + Amount\ of\ Sales\ Tax.$$

The next example illustrates this idea.

EXAMPLE 1 Sales tax in Florida is 6%. Compute the sales tax for a new LED television that sells for $350. What is the total cost of the LED TV?

SOLUTION: For the first part of this problem we are looking for the *Amount of Sales Tax*. The problem states that the *Tax Rate* is 6% and the *Cost of the Merchandise* is $350. Based on this information we can set up a percent proportion to find the *Amount of Sales Tax*. That is,

$$\frac{Amount\ of\ Sales\ Tax}{Cost\ of\ Merchandise} = Tax\ Rate \Rightarrow \frac{Amount\ of\ Sales\ Tax}{350} = 6\%.$$

Letting x = *Amount of Sales Tax* and converting 6% to a fraction we get

$$\frac{x}{350} = \frac{6}{100} \Rightarrow \frac{x}{350} = \frac{3}{50}.$$

Solving the percent proportion gives

$$\frac{x}{350} = \frac{21}{350}$$

so the *Amount of Sales Tax* is \$21.

The *Total Amount of the Purchase* is then the sum of the *Cost of the Merchandise* and the *Amount of Sales Tax*. Therefore, the total cost of the LED TV is $\$350 + \$21 = \$371$.

EXAMPLE 2 The sales tax on a bag of groceries came to \$1.25. If the sales tax rate is 5%, what was the price of the groceries before tax?

SOLUTION: In this problem we are looking for the *Cost of Merchandise*. We are given that the *Amount of Sales Tax* is \$1.25 and that the *Tax Rate* is 5%. Setting up the percent proportion gives

$$\frac{\textit{Amount of Sales Tax}}{\textit{Cost of Merchandise}} = \textit{Tax Rate} \Rightarrow \frac{1.25}{\textit{Cost of Merchandise}} = 5\%.$$

Letting x represent the *Cost of Merchandise* and converting 5% to a fraction we get

$$\frac{1.25}{x} = \frac{5}{100} \Rightarrow \frac{1.25}{x} = \frac{1}{20}.$$

Solving the percent proportion gives

$$\frac{1.25}{x} = \left(\frac{1}{20}\right)\left(\frac{1.25}{1.25}\right) = \frac{1.25}{25}$$

and so the cost of the groceries before tax is \$25.

EXAMPLE 3 Jocelyn has just moved to a new state and needs to buy a new dishwasher for her home. The dishwasher costs $258 and Jocelyn paid $272.19, which included tax. What is the sales tax rate?

SOLUTION: In this problem we are told that the cost of the dishwasher was $258 but that Jocelyn paid $272.19. This means that the *Amount of Sales Tax* is

$$\textit{Total Amount of Purchase} - \textit{Cost of Merchandise} = \$272.19 - \$258 = \$14.19.$$

Letting x represent the unknown *Tax Rate* we get the percent proportion

$$\frac{14.19}{258} = x\% \;\Rightarrow\; \frac{14.19}{258} = \frac{x}{100}.$$

Performing the division we get

$$\frac{x}{100} = 0.055$$

and so the *Tax Rate* is 5.5%.

6.4.2 Commission

Another common application involving percent is commission. Commission is a form of income based on a percent of sales and is calculated using the following percent proportion.

$$\frac{\textit{Amount of Commission}}{\textit{Amount of Sales}} = \textit{Commission Rate}$$

where *Commission Rate* is a percent.

If you have a job in which you earn commission, you can calculate your income as

$$\textit{Total Salary} = \textit{Base Salary} + \textit{Amount of Commission}.$$

The *Base Salary* is what you get paid without factoring in the commission you earn.

EXAMPLE 1 Jane earns a commission of 12% on sales over \$300. One day she sells \$650 worth of merchandise. How much did Jane make in commission that day?

SOLUTION: From the problem we know that the *Commission Rate* is 12% so

$$\frac{\textit{Amount of Commission}}{\textit{Amount of Sales}} = \textit{Commission Rate}$$

$$\Rightarrow \quad \frac{\textit{Amount of Commission}}{\textit{Amount of Sales}} = 12\%.$$

We are also told in the problem that she earns a commission on her sales over \$300. This means that if she sells \$650 worth of merchandise she will earn commission on

$$\$650 - \$300 = \$350.$$

So with *Amount of Sales* = \$350, the percent proportion becomes

$$\frac{\textit{Amount of Commission}}{350} = 12\%.$$

If we let x represent the unknown *Amount of Commission* and convert the percent to a fraction we have

$$\frac{x}{350} = \frac{12}{100}$$

$$\Rightarrow \quad \frac{x}{350} = \frac{3}{25}.$$

Since $350 \div 25 = 14$, we get

$$\frac{x}{350} = \left(\frac{3}{25}\right)\left(\frac{14}{14}\right) = \frac{42}{350}.$$

Therefore, Jane earns \$42 in commission.

EXAMPLE 2 Joseph is a real estate agent. His base salary is $16,000 per year but he also earns 10% commission on his total housing sales for the year. If his total salary this year was $123,000, what was his total in housing sales?

SOLUTION: We can calculate how much commission he earned on his housing sales by subtracting his base salary from his *Total Salary*. That is,

$$\textit{Amount of Commission} = \textit{Total Salary} - \textit{Base Salary}$$

$$= \$123,000 - \$16,000 = \$107,000.$$

We are also given that his *Commission Rate* is 10% so if we let x represent his total sales we get

$$\frac{\textit{Amount of Commission}}{\textit{Amount of Sales}} = \textit{Commission Rate}$$

$$\Rightarrow \quad \frac{107,000}{x} = 10\%.$$

Changing the percent to a fraction we have

$$\frac{107,000}{x} = \frac{1}{10}.$$

From this we see that

$$\frac{107,000}{x} = \left(\frac{1}{10}\right)\left(\frac{107,000}{107,000}\right) = \frac{107,000}{1,070,000}.$$

Therefore, Joseph sold a total $1,070,000 this year.

Section 6.4 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 5, fill in the missing information from the table.

	Cost of Merchandise	Tax Rate	Amount of Sales Tax	Total Cost
1)	\$40		\$2.40	
2)	\$300	5.5%		
3)		7%	\$6.16	
4)	\$1,200		\$108	
5)			\$2.34	\$54.34

For problems 6 - 16, assume the tax rate is 6%**. Round any answers to the nearest cent if needed.**

6) Jon buys a guitar for \$320. What is the amount of sales tax due?

7) Compute Jon's total bill for the \$320 guitar including sales tax.

8) Erin bought groceries for \$56.50. What is the amount of sales tax due?

9) What did Erin pay for the groceries worth \$56.50 including the tax?

10) Susan bought a used car and the taxes on the purchase totaled \$299.70. What was the price of her car before taxes were added?

11) What was the total cost of Susan's car including the tax?

12) Jordan paid \$8.25 in taxes when he returned a rental car. What was the charge for the rental car before taxes were added?

13) Determine Jordan's total bill for the rental car including the tax.

14) Lara bought a necklace with a price tag of \$60. If she used a 20% off discount coupon, what was the charge of the necklace after the 20% discount but before the taxes were added?

15) For Lara's necklace purchase, how much tax would be added to her bill?

16) What was Lara's overall cost for the necklace, after the discount with the taxes included?

For problems 17 - 20, determine the tax rate that is begin used.

17) Bertha bought a hat with a price tag of $23. The amount of sales tax was $1.61.

18) Randy bought a leather coat marked at $305. The amount of sales tax was $24.40.

19) Ben's bill for a $1200 piano was $1266 including tax.

20) Pablo bought a $17,000 motorcycle and paid $18,530 including tax.

For problems 21 - 25, fill in the missing information from the table.

	Total Sales	Rate of Commission	Amount of Commission
21)	$3,500	4%	
22)	$165,000	1.5%	
23)	$48,000		$1,680
24)		7%	$4,830
25)		2.25%	$2,216.25

For problems 26 - 34, solve the application involving commission.

26) Jose sold $3300 in merchandise last week and he earns a 3.5% commission on all of his sales. How much did he make in commission?

27) Dawn collected $780 in commission pay last week. If her commission rate is 3%, then what was the amount of sales Dawn made?

28) A realtor earned $13,000 on the sale of a $260,000 home. Determine the commission rate.

29) Frieda earns 4.5% on sales over $1000 for the week. Last week her sales totaled $1900. How much did Frieda make in commission last week?

30) Gordon makes $200 in base pay per week in addition to receiving a 5% commission on all of his sales. What is his pay for a week in which he has sales totaling $2500?

31) Rita sells fragrances and earns $350 in weekly salary as well as 4.5% of her total sales. She made $467 for the week. What was the total of her sales for the week?

32) A salesperson earns a monthly salary of $1000, plus a 6.5% commission on sales. She made $2885 in December. What was the amount of her sales in December?

33) Marie is a real estate agent who makes a base annual salary of $12,000 for her work in the office. Additionally, she receives a commission of 3.2% on her property sales. What were the total value of her sales in a year that her annual salary was $70,000?

34) Erica sells clothing in a department store. She earns an hourly wage of $7 and get a 6.5% commission on all merchandise that she sells over $300. On Wednesday, she worked for 10 hours and sold $710 worth of merchandise. How much will Erica earn for Wednesday?

Chapter 6
Section 6.4 – More Applications of Percent Proportions

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

1) Tax Rate: _____

Total Cost: _____

2) Amount: _____

Total Cost: _____

3) Cost: _____

Total Cost: _____

4) Tax Rate: _____

Total Cost: _____

5) Cost: _____

Tax Rate: _____

6) _____

7) _____

8) _____

9) _____

10) _____

11) _____

12) _____

13) _____

14) _____

15) _____

16) _____

17) _____

18) _____

19) _____

20) _____

21) _____

22) _____

23) _____

24) _____

25) _____

26) _____

27) _____

28) _____

29) _____

30) _____

31) _____

32) _____

33) _____

34) _____

GEOMETRY

CHAPTER

7

CHAPTER OBJECTIVES

7.1 Triangles and the Pythagorean Theorem

7.2 Perimeter and Area

7.3 Circles

7.4 Volume

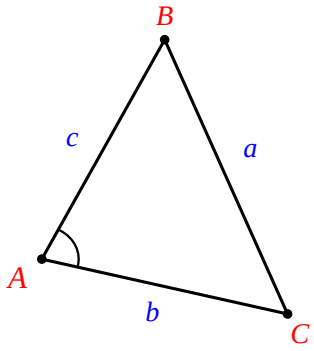
Steering Towards Efficiency

The racing team is just as important to a car's finish as the driver is. With little to separate competitors over hundreds of laps, teams search for any technological edge that will propel them to Victory Lane. Of special use today is computational fluid dynamics, which is used to predict airflow over a car, both alone and in relation to other cars (for example, when drafting). Engineers also rely on more basic subjects such as calculus and geometry, to improve their cars. In fact, one racing team engineer said of his calculus and physics teachers, "the classes they taught to this day were the most important classes I've ever taken."

Mathematics helps the performance and efficiency of non-NASCAR vehicles, as well. To improve engine performance, data must be collected and processed very rapidly so that control devices can make adjustments to significant quantities such as air/fuel ratios. Innovative sampling techniques make this real-time data collection and processing possible. This makes for lower emissions and improved fuel economy--goals worthy of a checkered flag.

7.1 Triangles and the Pythagorean Theorem

A **triangle** is a three sided, closed, planar figure. They can be classified, or categorized, according to the lengths of their sides or by the measure of their internal angles. For reference, consider the triangle



Quick Note

It is common to use capital letters to denote vertices and lower case letter to represent sides.

The sides of the triangle meet at points called **vertices**. In the figure, the vertices of the triangle are labelled A, B, and C. As a result we would refer to this triangle as $\triangle ABC$.

Notice that the vertices are denoted with upper case letters and the sides of the triangle opposite the vertices are labeled with lower case letters. For the figure, the side opposite vertex A is labeled as a, the side opposite vertex B is labeled as b, and the side opposite vertex C is labeled as c.

Also shown in the figure is the **angle** formed by sides b and c meeting at vertex A. Because the angle occurs at the vertex A we refer to it as *angle A* and we write $\angle A$. The subject of angles is where we begin a more detailed discussion of triangles.

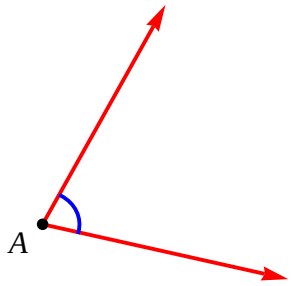
7.1.1 Angles

A **ray** is a directed line segment, or arrow. An **angle** is formed when two rays are drawn from a common vertex.

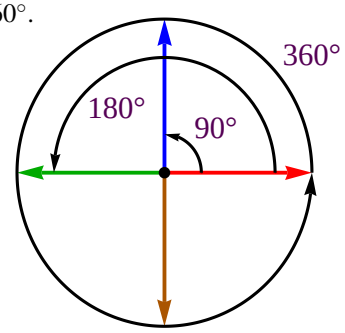
Quick Note

Lines do not have a finite length but a line segment does.

The triangle shown above has three internal angles. The internal angle at vertex A is formed by the two sides of the triangle. The sides of the triangle forming $\angle A$ are not *rays*. The $\angle A$ shown by itself, not as part of the triangle, would appear as shown in the following figure with rays as its two sides.



The size of an angle is measured in degrees. We base the measure of angles off a circle which consists of 360 degrees, or 360°.



In the figure above, if we start on the red ray and travel counterclockwise one full circle and end up back on the red ray we have traveled a total of 360°.

If we start on the red ray and travel half way around the circle to the green ray we have traveled a total of 180° since $\frac{360^\circ}{2} = 180^\circ$.

If we start on the red ray and travel one fourth of the way around the circle to the blue ray we have traveled a total of 90° since $\frac{360^\circ}{4} = 90^\circ$.

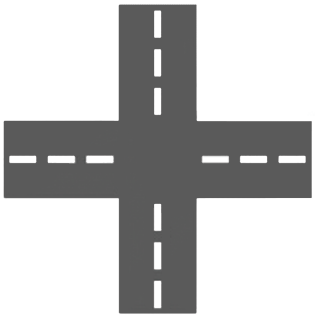
We define types of angles based on their size relative to 90° and 180° as discussed in the next section.

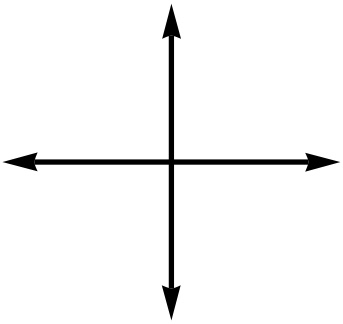
We use the idea of rays in everyday life and future courses in the form of lines. While a ray, a directed line segment, has a beginning "point" and a direction (the direction it is pointing), lines do not have a starting point because they go on forever. However, when two or more lines are shown together they do have direction similar to rays. As an example, we can use parallel and perpendicular lines to approximate traffic lanes and intersections.

Perpendicular lines are lines such that when they intersect they form four equal angles. Most street intersections are formed by two perpendicular roads.

Quick Note

The ° symbol can be used for temperature or for measuring angles. The context of the problem tells us whether temperature or angle measure is implied by the symbol.

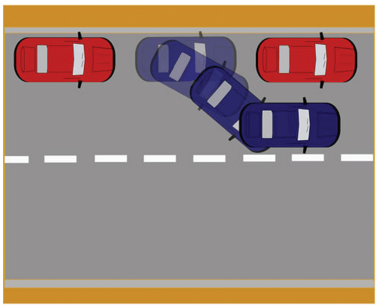


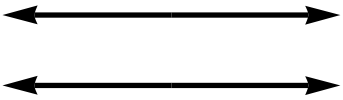


Quick Note

Sometimes the symbol $\perp$ is used to denote perpendicular lines.

Two lines are parallel if they never intersect one another. The term "parallel parking" takes its name from the fact that the parking spot is parallel to the traffic lane.





Quick Note

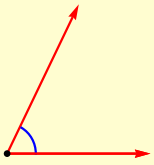
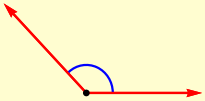
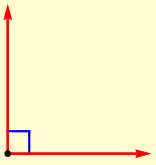
Sometimes the symbol $||$ is used to denote parallel lines.

7.1.2 Types of Angles

We say that an angle is **acute** if its measure is between 0° and 90°.

We say that an angle is a **right angle** if its measure is exactly 90°.

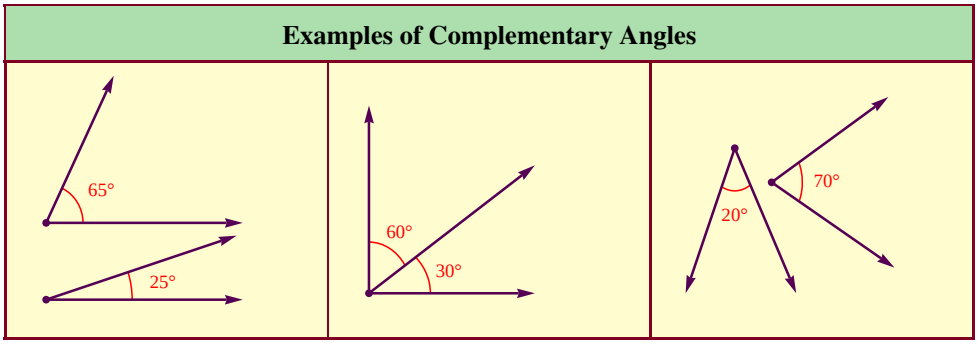
An angle is **obtuse** if its measure is between 90° and 180°.

Typical Acute Angle	Typical Obtuse Angle	Right Angle
		

Pairs of Angles - Complementary and Supplementary Angles

Two angles are **Complementary Angles** if the sum of their measures equals $90°$. As an example, the angles $60°$ and $30°$ are complementary because $60° + 30° = 90°$.

Examples of Complementary Angles



EXAMPLE 1 Find the measure of the complement of the angle $53°$.

SOLUTION: The complement of the angle $53°$ is an angle α whose measure is such that

$$53° + \alpha = 90°.$$

To determine the measure of α, subtract the known angle from $90°$. That is,

$$\alpha = 90° - 53° = 37°.$$

Quick Note

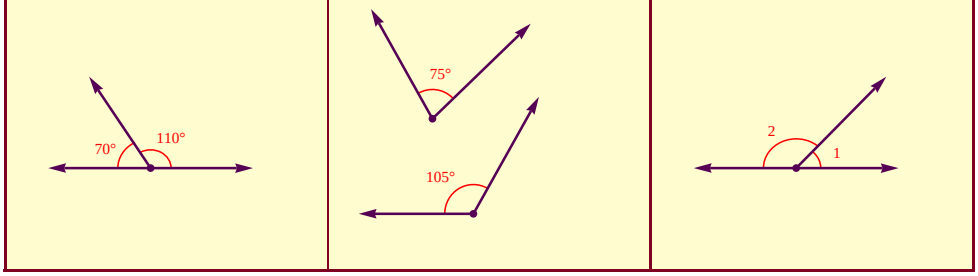
To remember the difference between complementary and supplementary, think

complementary $\Leftrightarrow$ corner

supplementary $\Leftrightarrow$ straight

Two angles are **Supplementary Angles** if the sum of their measures equals $180°$. As an example, the angles $78°$ and $102°$ are supplementary because $78° + 102° = 180°$.

Examples of Supplementary Angles



EXAMPLE 2 Find the measure of the supplement of the angle $109°$.

SOLUTION: The supplement of 109° is an angle β such that

$$109^\circ + \beta = 180^\circ.$$

To determine the measure of β, subtract the measure of the known angle form 180°. That is,

$$\beta = 180^\circ - 109^\circ = 71^\circ.$$

Therefore, the supplement of 109° is 71°.

7.1.3 Types of Triangles

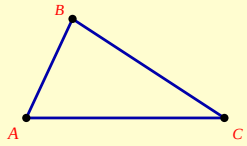
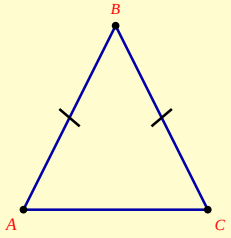
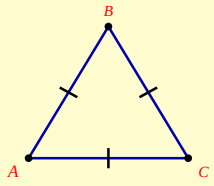
There are two ways in which a triangle can be classified. They can be classified according to the lengths of their sides or by their internal angles. The two types of classification are not always distinct.

Triangles Classified by Side Length

A triangle is called a **scalene triangle** if no sides are congruent (equal in length).

A triangle is called an **isosceles triangle** if two sides are congruent.

A triangle is called an **equilateral triangle** if all three sides are congruent.

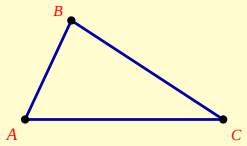
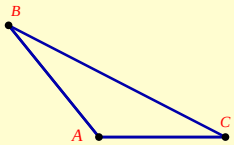
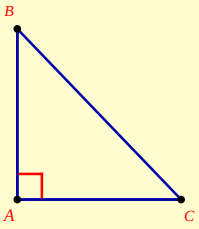
Scalene Triangle	Isosceles Triangle	Equilateral Triangle
		

Triangles Classified by Measure of Internal Angles

A triangle is called an **acute triangle** if all of the internal angles are acute. Note that the scalene triangle shown above is also an acute triangle.

A triangle is called an **obtuse triangle** if one of its internal angles is obtuse. Note that if one of the internal angles is obtuse the other two must be acute.

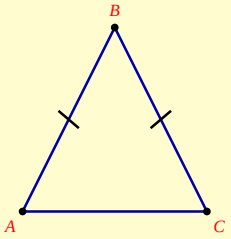
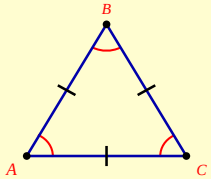
A triangle in which one angle is a right angle is called a **right triangle**.

Acute Triangle	Obtuse Triangle	Right Triangle
		

For a right triangle we call the side opposite the right angle (the long side of the right triangle) the **Hypotenuse**. The other two sides of a right triangle are referred to as the **legs**.

Connection Between Side Length and Measure of Internal Angles

There is a connection between the length of the sides of a triangle and the measure of the internal angles. The connection comes into play with Isosceles and Equilateral Triangles.

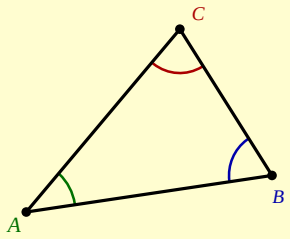
Isosceles Triangle	Equilateral Triangle
	

In an Isosceles Triangle, the two base angles are congruent (they have the same measure). In an Equilateral Triangle, all internal angles are congruent.

7.1.4 Triangle Postulate

The geometry studied in this course is called Euclidean Geometry. Any triangle in Euclidean Geometry must satisfy a certain property called the Triangle Postulate.

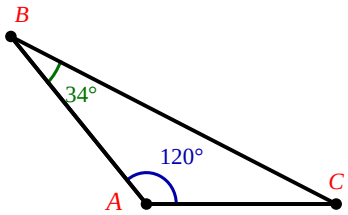
Triangle Postulate - Interior Angle Sum



In any triangle

$$m\angle A + m\angle B + m\angle C = 180^\circ.$$

EXAMPLE 1 Find the measure of the missing angle in the triangle.



SOLUTION: Because the sum of the measures of the internal angles must equal 180° we know that

$$m\angle A + m\angle B + m\angle C = 180^\circ.$$

We know $m\angle A = 120^\circ$ and $m\angle B = 34^\circ$ so we can write this as

$$120^\circ + 34^\circ + m\angle C = 180^\circ.$$

$$\Rightarrow \quad 154^\circ + m\angle C = 180^\circ.$$

To find $m\angle C$ we need only subtract the measures of the known angles from 180°. This gives

$$m\angle C = 180^\circ - 154^\circ = 26^\circ.$$

7.1.5 Right Triangles and the Pythagorean Theorem

A right triangle is a triangle in which one of the internal angles measures exactly $90°$. The other two internal angles are acute.

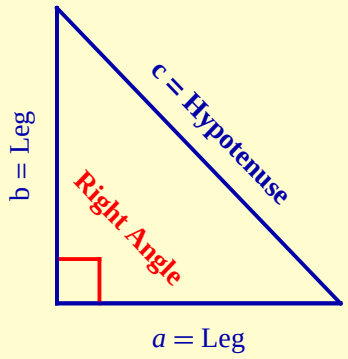
The side of the triangle opposite the right angle (the $90°$ angle) is called the **hypotenuse** and is typically denoted as c. The other two sides are called the **legs** and are denoted as a and b.

What makes right triangles special is the relationship between the lengths of its legs and its hypotenuse. This relationship is called the **Pythagorean Theorem**.

Pythagorean Theorem

In any right triangle, if the hypotenuse is denoted as c and the legs are denoted as a and b, then the Pythagorean Theorem states that

$$a^2 + b^2 = c^2$$



To use the Pythagorean Theorem we need to be able to find what's called the **Square Root** of a number.

Square Roots

Square Root
For any positive real number a, the square root of a is a real number b such that $b^2 = a$.
The square root of a is denoted as $\sqrt{a}$.

The symbol $\sqrt{\ }$ is called a **radical**. Moreover, in the expression $\sqrt{a}$, we refer to a as the **radicand**.

To calculate a square root we can apply the technique of section 3.2. That is, we can calculate square roots using prime factorizations.

Remember that a number is prime if it is only divisible by itself and one.

EXAMPLE 1 Simplify $\sqrt{144}$.

SOLUTION: The prime factorization of 144 is

$$144 = 2 \cdot 72$$

$$= 2 \cdot 2 \cdot 36$$

$$= 2 \cdot 2 \cdot 2 \cdot 18$$

$$= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 9$$

$$= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$$

Notice that if we regroup the prime factors into groups of two factors at a time we have

$$144 = (2 \cdot 2)(2 \cdot 2)(3 \cdot 3) = 2^2 \cdot 2^2 \cdot 3^2.$$

Again, regroup and write this as

$$144 = (2 \cdot 2 \cdot 3)^2 = 12^2.$$

Now because 12 is the number such that $12^2 = 144$,

$$\sqrt{144} = 12.$$

Because we were looking for a square root and the square root of a number a is defined to be the number b such that $b^2 = a$, we grouped the factors of the number a into groups of 2.
In courses after this one you will be asked to calculate different "roots." Here we are concerned only with square roots.

EXAMPLE 2 Simplify $\sqrt{900}$.

SOLUTION: The prime factorization of 900 is

$$900 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5.$$

Grouping factors in pairs and regrouping gives

$$900 = (2 \cdot 2)(3 \cdot 3)(5 \cdot 5) = (2 \cdot 3 \cdot 5)^2 = 30^2.$$

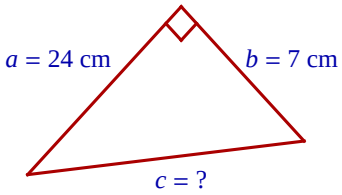
Therefore, $\sqrt{900} = 30$.

It should be pointed out that the numbers for which you will be asked to find the square root of, will be numbers called "perfect squares." A perfect square is a rational number that is the square of another rational number. You will take future course in which the numbers you must take the square root of are not perfect squares.

Pythagorean Theorem

EXAMPLE 3 A right triangle has legs of length 24 cm and 7 cm. Find the length of the hypotenuse.

SOLUTION: For these problems you may find it helpful to draw a triangle with the given dimensions. The important thing to remember when drawing any right triangle is that the hypotenuse is the side opposite the right angle. For this problem we know the lengths of the legs but we need to find the length of the hypotenuse.



Once we have identified that $a = 24$ cm, $b = 7$ cm, and the hypotenuse, c, is unknown we can use the Pythagorean Theorem. That is,

$$a^2 + b^2 = c^2 \;\Rightarrow\; (24)^2 + (7)^2 = c^2.$$

Simplifying we get

$$c^2 = 576 + 49 = 625.$$

This says that c is the number such that $c^2 = 625$, therefore, $c = \sqrt{625}$.

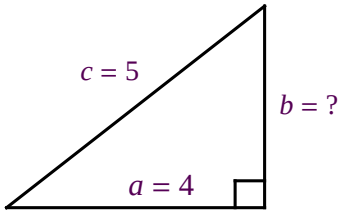
The prime factorization of 625 is

$$625 = 5 \cdot 5 \cdot 5 \cdot 5 = (5 \cdot 5)^2 = 25^2$$

and we see the length of the hypotenuse is 25 cm.

EXAMPLE 4 A right triangle has a leg of length 4 and a hypotenuse of length 5. Find the length of the other leg of the triangle.

SOLUTION: For this problem we have $c = 5$, $a = 4$, and b unknown.



Using the Pythagorean Theorem we have

$$a^2 + b^2 = c^2 \;\Rightarrow\; 4^2 + b^2 = 5^2.$$

$$\Rightarrow\; 16 + b^2 = 25.$$

To figure out what to add to 16 to get 25 we can subtract 16 from 25. That is,

$$b^2 = 25 - 16 = 9.$$

Since $3^2 = 9$, the length of the other side of the right triangle is 3.

Section 7.1 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 4, find the measure of the complement of the given angle.

1) $40°$ 2) $62°$ 3) $14°$ 4) $78°$

For problems 5 - 8, find the measure of the supplement of the given angle.

5) $130°$ 6) $107°$ 7) $52°$ 8) $72°$

For problems 9 - 11, answer each question in your own words.

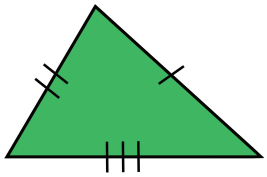
9) What is an acute angle?

10) What is an obtuse angle?

11) What is a right angle?

For problems 12 - 14, label the triangle as either scalene, isosceles, or right.

12)



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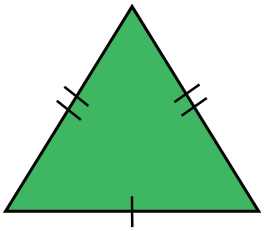

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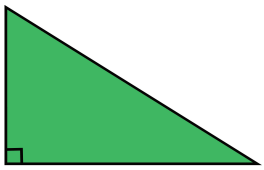
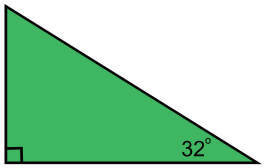


For problems 15 and 16, find the measure of the missing angle in the triangle.

15)



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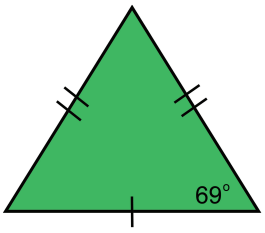


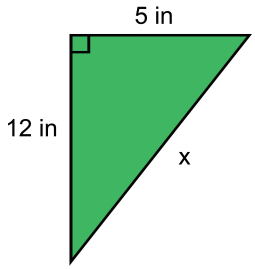
For problems 17 - 19, simplify the given square root.

17) $\sqrt{25}$ 18) $\sqrt{81}$ 19) $\sqrt{121}$

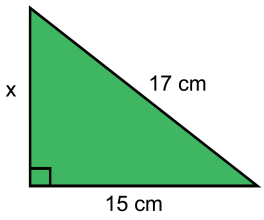
20) State the Pythagorean Theorem.

For problems 21 - 23, use the Pythagorean Theorem to find the missing side of the triangle.

21)



22)



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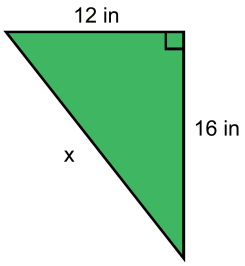

24) If the hypotenuse of a triangle is 15 and one leg is 12, find the length of the other leg.

25) A UPS delivery truck travels 10 miles west and 24 miles north. How far is it from its starting point?

Review. Simplify the given expression.

26) $3+(-12)$

27) $-2-(-6)$

28) $-7(-8)$

29) $15 \div (-5)$

30) $\frac{5}{9}+\frac{5}{9}$

31) $\frac{7}{11}-\frac{2}{11}$

32) $\frac{2}{5}+\frac{1}{3}$

33) $\frac{3}{4}-\frac{1}{3}$

34) $\frac{5}{3} \times \frac{6}{7}$

35) $\frac{1}{2} \div \frac{2}{3}$

Chapter 7
Section 7.1 – Triangles and the Pythagorean Theorem

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

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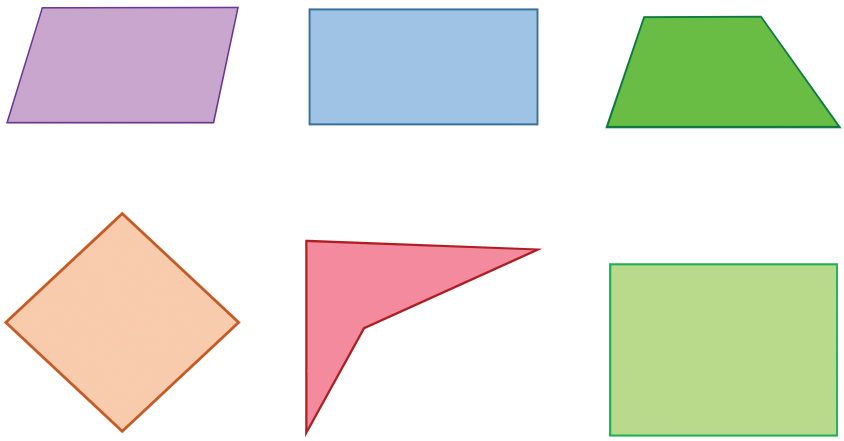
7.2 Perimeter and Area

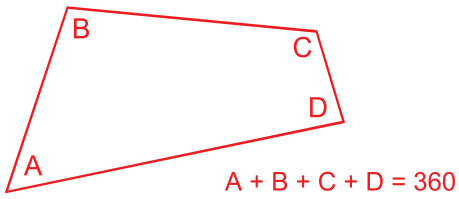
In this section we revisit the subject of finding the perimeter and area of certain figures. Before we begin looking at those applications we define some common shapes much more rigorously than we have in the past.

7.2.1 Quadrilaterals

Similar to the definition of a triangle, a **quadrilateral** is a four-sided, closed, planar figure. A few examples are shown here.

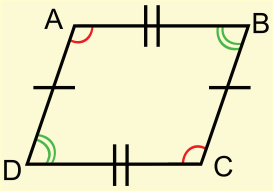


In every quadrilateral, there are exactly four sides and the sum of the interior angles is 360°.



There are some specific quadrilaterals that we work with often. Their properties are summed up in the following.

Properties of Parallelograms



1) Opposite sides are parallel: $\overline{AD}||\overline{BC}, \overline{AB}||\overline{DC}$

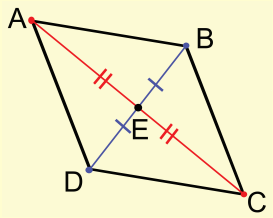
2) Opposite sides are congruent: $\overline{AD} = \overline{BC}$, $\overline{AB} = \overline{DC}$

3) Opposite angles are congruent: $m\angle A = m\angle C$, $m\angle D = m\angle B$.

4) Consecutive angles are supplementary:

$$m\angle A + m\angle B = 180^\circ, \quad m\angle B + m\angle C = 180^\circ$$

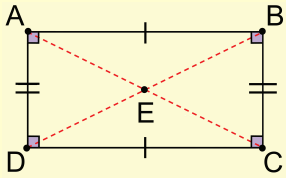
$$m\angle C + m\angle D = 180^\circ, \quad m\angle D + m\angle A = 180^\circ$$



5) Diagonals bisect each other.

6) Diagonals form two congruent triangles.

Properties of Rectangles

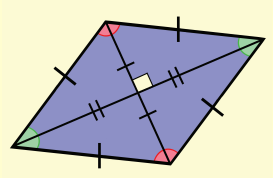


Rectangles are parallelograms with a few additional properties.

1) All internal angles are right angles.

2) Diagonals are congruent.

Properties of a Rhombus



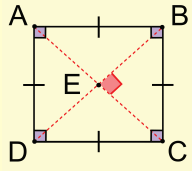
A Rhombus is a parallelogram with a few additional properties.

1) All four sides are congruent.

2) Diagonals bisect each other.

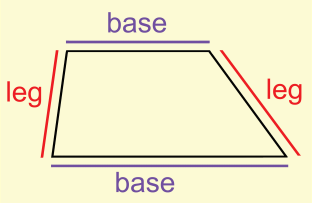
3) Diagonals are perpendicular to each other.

Properties of a Square



A Square has all of the properties of a parallelogram, rectangle, and rhombus.

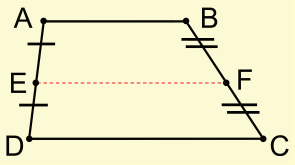
Properties of a Trapezoid



1) Has only one set of parallel sides.

2) The median of a trapezoid is parallel to the bases and equal to one-half the sum of the bases.

Properties of an Isosceles Trapezoid



1) Has one set of parallel sides

2) The base angles are congruent

$$m\angle D = m\angle C \quad m\angle A = m\angle B$$

3) The legs are congruent

$$\overline{AD} = \overline{BC}$$

4) The diagonals are congruent

$$\overline{AC} = \overline{BD}$$

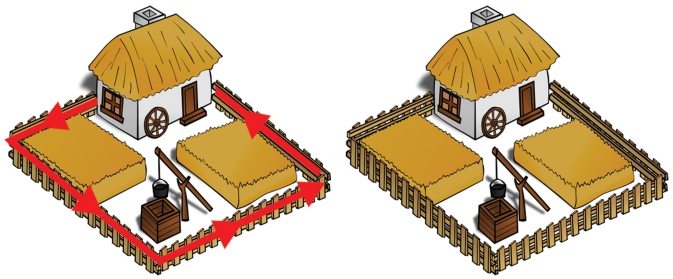
5) Opposite angles are supplementary

$$m\angle A + m\angle C = 180^\circ \quad m\angle B + m\angle D = 180^\circ$$

Perimeter and Area of a Polygon

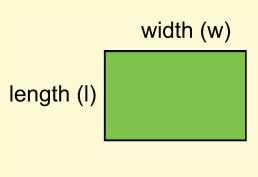
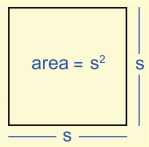
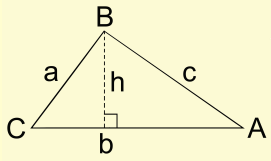
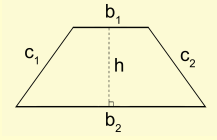
We have encountered perimeter and area in previous sections and applications. We revisit the concepts here.

Recall, the perimeter of a polygon is the distance around the outside of the polygon. For an example, consider the picture of the yard shown here. The perimeter of the yard is the amount of fencing needed to enclose it.



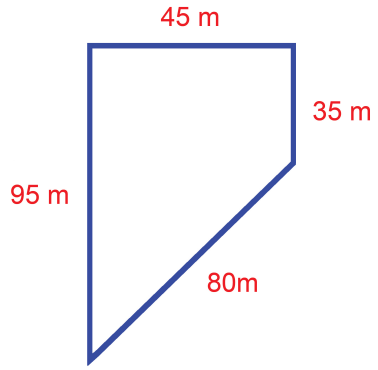
The perimeter of a polygon can also be thought of as the distance it would take to walk around the border of the polygon. This would mean that the perimeter of the yard shown here is the distance it would take to walk around the yard on the fence. One thing to remember is that perimeter is a linear measurement.

The **area** of a polygon is the number of square units that it encloses. This means that area is given in square units. In this section we will be working with general polygons as well as some common polygons that have known formulas for calculating their perimeter and area. The formulas for the common polygons are given here.

Perimeter and Area of Common Polygons			
Rectangle	**Square**	**Triangle**	**Trapezoid**
			
Perimeter: $P = 2l + 2w$ Area: $A = lw$	Perimeter: $P = 4s$ Area: $A = s^2$	Perimeter: $P = a + b + c$ Area: $A = \frac{1}{2}bh$	Perimeter: $P = b_1 + c_1 + b_2 + c_2$ Area: $A = \frac{1}{2}(b_1 + b_2)h$

The examples illustrate how to find the perimeter of general polygons and how to find the area of the common polygons using the formulas given above.

EXAMPLE 1 Find the perimeter of the polygon shown.

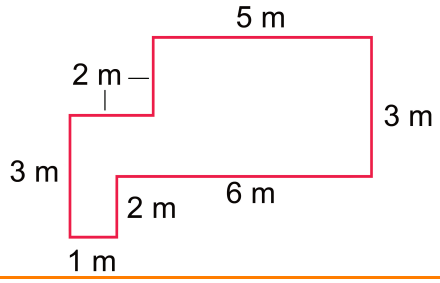


SOLUTION: To find the perimeter we need to add the lengths of all the sides. Therefore,

$$\text{Perimeter} = 95\,\text{m} + 45\,\text{m} + 35\,\text{m} + 80\,\text{m}$$

$$\Rightarrow \text{Perimeter} = 255\,\text{m}$$

EXAMPLE 2 Find the perimeter of the polygon shown.



SOLUTION: To find the perimeter we need to add the lengths of all the sides. Therefore,

$$\text{Perimeter} = 3\,\text{m} + 2\,\text{m} + 5\,\text{m} + 3\,\text{m} + 2\,\text{m} + 1\,\text{m}$$

$$\Rightarrow \text{Perimeter} = 16\,\text{m}$$

EXAMPLE 3 Find the perimeter and area of the rectangle shown.

SOLUTION: To find the perimeter of any polygon we need to add up the lengths of all the sides. For this rectangle we have

$$P = 12\,\text{cm} + 7\,\text{cm} + 12\,\text{cm} + 7\,\text{cm}$$

$$= 38\,\text{cm}.$$

The formula for the perimeter of a rectangle is $P = 2l + 2w$ where l represents the length of the rectangle and w represents the width. So with $l = 12\,\text{cm}$ and $w = 7\,\text{cm}$, the formula gives

$$P = 2(12\,\text{cm}) + 2(7\,\text{cm})$$

$$= 24\,\text{cm} + 14\,\text{cm}$$

$$= 38\,\text{cm}.$$

To find the area of the rectangle we use the formula.

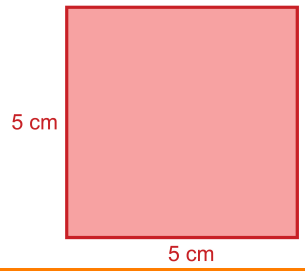
$$A = lw$$

$$\Rightarrow\ A = (12\,\text{cm})(7\,\text{cm})$$

$$= (12)(7)\,\text{cm}^2$$

$$= 84\,\text{cm}^2.$$

EXAMPLE 4 Find the perimeter and area of the square shown.



SOLUTION: To find the perimeter of any polygon, sum the lengths of all the sides. Here,

$$P = 5\,\text{cm} + 5\,\text{cm} + 5\,\text{cm} + 5\,\text{cm} = 20\,\text{cm}.$$

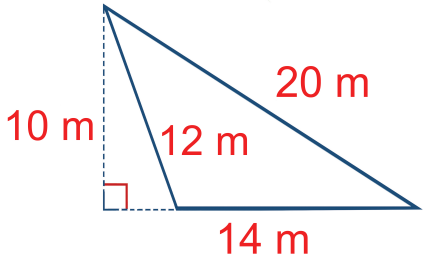
We could have also used the formula for the perimeter of a square. The formula is $P = 4s$ where s represents the length of the side of the square. So with $s = 5\,\text{cm}$, the formula gives

$$P = 4(5\,\text{cm}) = 20\,\text{cm}.$$

To find the area of the square we use the formula.

$$A = s^2 = (5\,\text{cm})^2 = 25\,\text{cm}^2.$$

EXAMPLE 5 Find the perimeter and area of the triangle shown.



SOLUTION: To find the perimeter of any polygon we need to add up the lengths of all the sides. For this triangle we have

$$P = 14\,\text{m} + 12\,\text{m} + 20\,\text{m} = 46\,\text{m}.$$

Notice that the formula for the perimeter of a triangle is $P = a+b+c$ where a, b, and c represent the lengths of the sides of the triangle. This is exactly what we calculated just based on the definition of Perimeter alone.

To find the area of the triangle we use the formula

$$A = \frac{1}{2}bh$$

where b represents the length of the base of the triangle and h represents the perpendicular height.

The perpendicular height of the triangle should make a right angle with its base. In the figure we see that $h = 10\,\text{m}$ and $b = 14\,\text{m}$. Therefore, the area is

$$A = \frac{1}{2}bh = \frac{1}{2}(14\,\text{m})(10\,\text{m}) = 70\,\text{m}^2.$$

Section 7.2 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

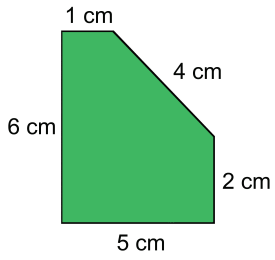
For problems 1 - 5, provide a definition for each quadrilateral.

1) Square 2) Parallelogram 3) Rectangle 4) Rhombus 5) Trapezoid

6) Describe how to find the perimeter of any polygon.

For problems 7 - 9, find the perimeter of the given polygon.

7)



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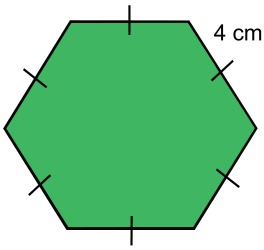

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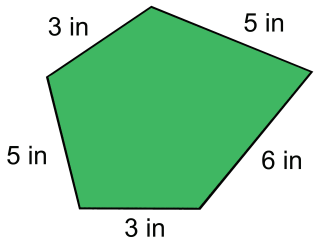


10) Find the perimeter of a rectangle with length 8 m and width 3 m.

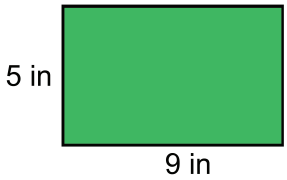
11) Find the perimeter of a rectangle with length 2 cm and width 50 cm.

12) Find the perimeter of a triangle with sides 3 in, 10 in and 7 in.

13) Describe how to find the area of a triangle.

For problems 13 - 19, find the area of the given polygon.

14)



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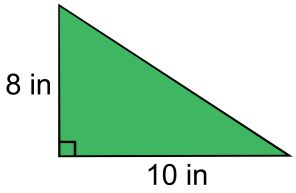

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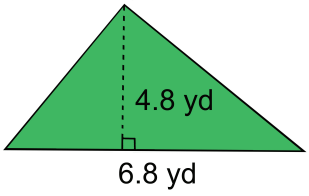

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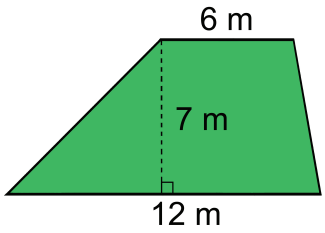

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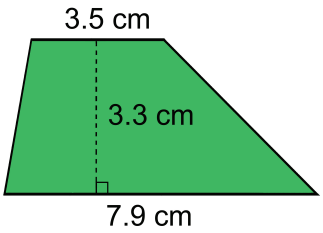

Mixed Practice

19) Find the complement of a 45° angle.

20) Is a 100° angle acute, right, or obtuse?

21) Two of the angles in a triangle are 20° and 100°, find the third angle.

22) A right triangle has a hypotenuse of 41 cm and one leg that is 9 cm. Find the length of the other leg.

Review. Perform the indicated operation(s).

23) $17.1 + 3.2$

24) $18.5 + 42.71$

25) $9.57 - 3.81$

26) $15 - 3.04$

27) $-3.4 - (-7.1)$

28) $5(4.2)$

29) $-0.11(-2.5)$

30) $5.79 \div 0.001$

31) $6.2 \div -0.31$

32) $0.5 \div 5$

33) $\left(\frac{3}{5}\right) \div \left(-\frac{10}{9}\right)$

Chapter 7
Section 7.2 – Perimeter and Area

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

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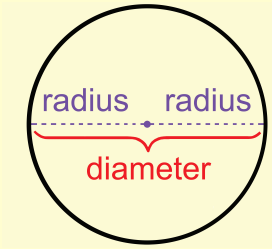
7.3 Circles

A **circle** is the set of all points that are the same distance away from a given point. The "same distance" is the **radius** of the circle, and the "given point" is the **center** of the circle.



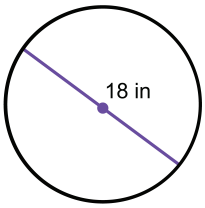
Radius: The radius of a circle is the length of the line segment with one endpoint on the circle and the other endpoint at the center of the circle.

Diameter: The diameter of a circle is the length of the line segment that passes through the center of the circle and whose endpoints are on the circle.

Make the observation that the diameter of a circle is twice the length of the radius. So if we let r denote the radius and d denote the diameter,

$$d = 2r.$$

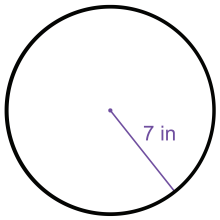
EXAMPLE 1 Find the radius of the circle shown.



SOLUTION: From the figure we see that the diameter is 18 in. Since the diameter is twice the length of the radius, the radius is one-half the length of the diameter. Therefore,

$$r = \frac{1}{2}d = \frac{1}{2}(18\,\text{in}) = 9\,\text{in}.$$

EXAMPLE 2 Find the diameter of the circle shown.

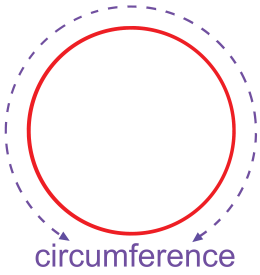


SOLUTION: From the figure we see that the radius is 7 in. Since the diameter is twice the length of the radius,

$$d = 2r = 2(7\,\text{in}) = 14\,\text{in}.$$

Circumference

The perimeter, or the distance around, a circle is known as the **circumference**. The reason we do not use the term perimeter when referring to the distance around a circle is that perimeter is used to describe the distance around a polygon. Circles are not polygons.



Determining the circumference of a circle is a problem that began thousands of years ago. In fact many ancient mathematicians and philosophers attempted to approximate the circumference of circles with regular polygons. One of the most important constants in mathematics was discovered by those investigating the circumference of a circle. It was found that the ratio of a circles circumference to its diameter is a constant regardless of the size of the circle. This constant is known as **pi** and is denoted as π.

If C denotes the circumference of a circle and d denotes the diameter, this constant π is given by

$$\pi = \frac{C}{d}.$$

We can manipulate this relationship to get a formula that can be used to find the circumference of a circle. That is, $C = \pi d$.

Since the diameter is twice the length of the radius we can rewrite this formula in terms of the radius. That is,

$$C = \pi d = \pi (2r) \quad \text{or} \quad C = 2\pi r.$$

The number π is an example of what are called **Irrational Numbers**. In chapter 3 we worked with Rational Numbers. We mentioned then that Rational Numbers are numbers that can be written as fractions. Since Rational Numbers can be written as fractions, their decimal representations are either terminating or repeating. Irrational Numbers are numbers that cannot be written as fractions and therefore their decimal representations are neither terminating nor repeating. To this day computer programmers and mathematicians compete to calculate more places in the decimal representation of the number π.

Because the irrational number π cannot be expressed as a fraction we technically will never know its exact value. To twenty decimal places

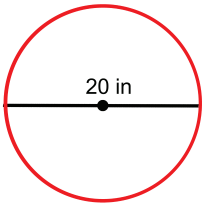
$$\pi \approx 3.14159265358979323846.$$

This would be cumbersome to have to remember every time we want to perform a calculation involving π, so many times we use a Rational Number as an approximation. Some common approximations to π are

$$\pi \approx 3.14, \ \pi \approx \frac{22}{7}, \quad \text{or} \quad \pi \approx \frac{355}{113}.$$

We also note that really any calculator that costs more than \$5 has a π key.

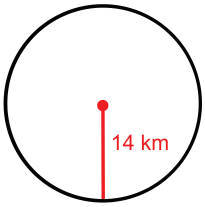
EXAMPLE 1 Find the circumference of the circle shown. Use 3.14 as a approximation for π. Round your answer to the nearest tenth.



SOLUTION: Since we are given the diameter of the circle we would use the formula $C = \pi d$, to determine its circumference. Therefore,

$$C = \pi d \Rightarrow C \approx 3.14(20\,\text{in}) = 62.8\,\text{in}.$$

EXAMPLE 2 Find the circumference of the circle shown. Use 3.14 as a approximation for π. Round your answer to the nearest tenth.



SOLUTION: Since we are given the radius of this circle,

$$C = 2\pi r \Rightarrow C \approx 2(3.14)(14\,\text{km}) = 87.9\,\text{km}.$$

Area

Finding the area of a circle is another problem with a rich history. An ancient problem in mathematics is known as "squaring the circle."

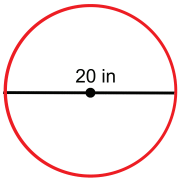
The problem involved trying to construct a square with the same area as a circle using only a compass and a straight edge. It wasn't until the 1880's that it was proved to be an impossible task. It was found that the area of a circle is given by the formula

$$A = \pi r^2, \text{ where } r \text{ represents the radius.}$$

Quick Note

To express the formula for the circumference of a circle we can use either the radius ($C = 2\pi r$) of the diameter ($C = \pi d$). To express the formula for the area of a circle we will always use the radius ($A = \pi r^2$).

EXAMPLE 1 Find the area of the circle shown. Use 3.14 as a approximation for π. Round your answer to the nearest tenth if appropriate.



SOLUTION: To find the area of a circle we need to know its radius.

In this problem we are given the diameter so we need to calculate the radius before we can calculate the area.

Since the radius is one-half of the diameter we know that

$$r = \frac{1}{2}(10 \text{ in}) = 10 \text{ in.}$$

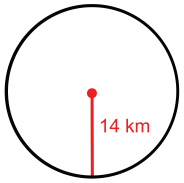
Plugging this into the formula for the area of a circle we get

$$A = \pi r^2$$

$$\Rightarrow A \approx 3.14(10\text{in})^2$$

$$= 314.0\text{in}^2.$$

EXAMPLE 2 Find the area of the circle shown. Use 3.14 as a approximation for π. Round your answer to the nearest tenth.



SOLUTION: Since we are given the radius of this circle we can use the formula to find the area of the circle.

So for this problem,

$$A = \pi r^2$$

$$\Rightarrow A \approx 3.14(14 \text{ km})^2$$

$$= 615.44 \text{ km}^2.$$

Rounding this to the nearest tenth we get that the area is 615.4 km^2.

Section 7.3 Exercises

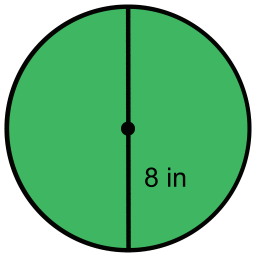
Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 2, provide a definition for each term.

1) Radius of a circle 2) Diameter of a circle.

For problems 3 - 4, find the radius of the circle shown.

3)



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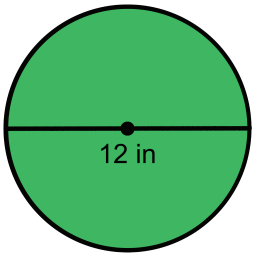
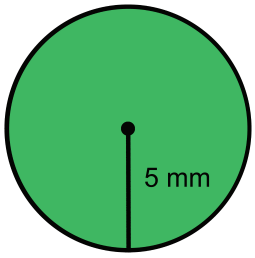


5) A penny has a diameter of 0.75 inches. What is its radius?

For problems 6 - 7, find the diameter of the circle shown.

6)



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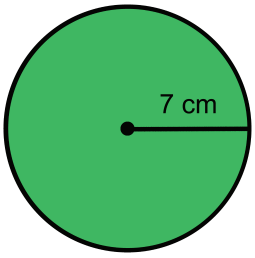
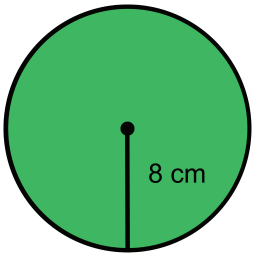


8) A quarter has a radius of 12.13 mm. What is its diameter?

9) What does the circumference of a circle represent? 10) What is the formula for the circumference of a circle?

For problems 11 - 15, find the circumference of the circle shown. Use $\pi \approx 3.14$.

11)



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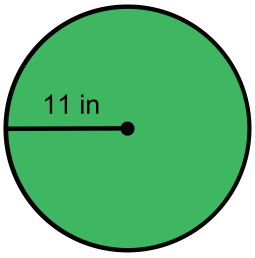

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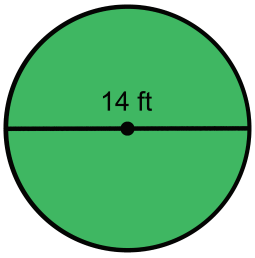
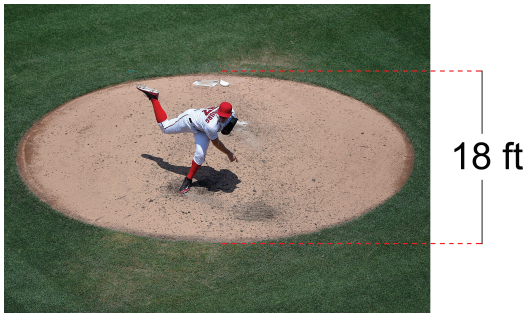
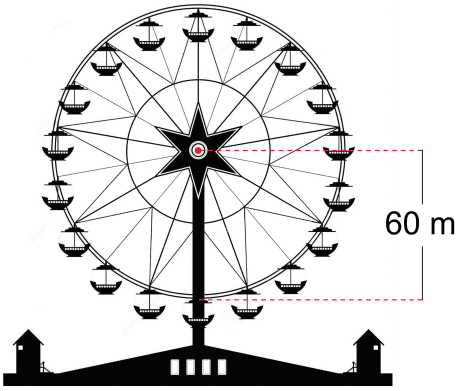


14) A pitcher's mound is a circle with diameter 18 feet. What is its circumference?



15) The London Eye is a Ferris Wheel with a radius of 60 meters. What is its circumference?

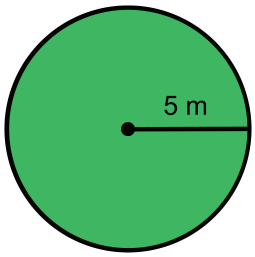


16) What is the formula for the area of a circle?

17) A circular fence has a radius of 30 feet. What is its area?

For problems 18 - 21, find the area of the circle shown. Round answers to the hundredths if needed. Use $\pi \approx 3.14$.

18)



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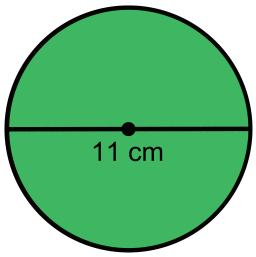

20) Find the area of a quarter (quarters have a radius of 12.13 mm).

21) Find the area of a penny (pennies have a diameter of 0.75 in).

Mixed Practice.

22) Find the perimeter of a square with side length 12 in.

23) Find the area of a triangle with base 9 feet and height 4 feet.

24) If two angles of a triangle are 30^0 and 75^0, what is the third angle?

Review.

25) Simplify: $\sqrt{121}$

26) Convert 30% to a decimal.

27) Convert 6.3% to a decimal.

28) Convert 0.15 to a percentage.

29) Convert 0.095 to a percentage.

30) Find 25% of 36.

31) 51 is what percent of 85?

32) $35 in groceries are purchased. If the sales tax is 6%, what is the total purchase price?

33) Sydney earns 15% commission on her sales. If her sales totaled $\$18,000$ for the month, what is her commission?

34) 120 students are surveyed and 15% of them are interested in a career in Information Technology. What is the total number of students interested in this career?

35) Alex purchases a television for $\$247.25$. If the cost before tax is $\$230$, then what is the sales tax rate?

Chapter 7
Section 7.3 - Circles

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

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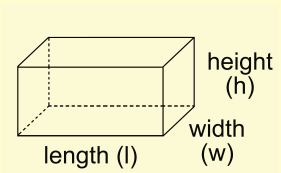
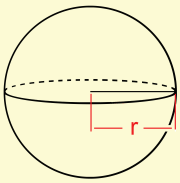
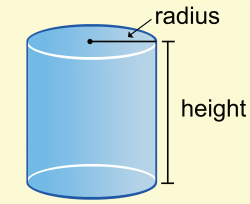
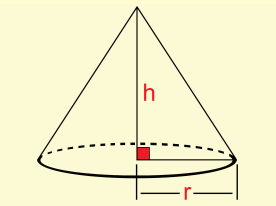
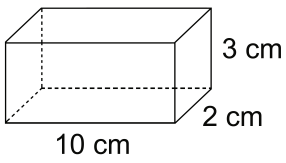
7.4 Volume

Volume is the amount of 3-dimensional space enclosed or occupied by a closed 3-dimensional object. Volume is measured in cubic units such as cubic inches (in^3), cubic meters (m^3), etc.

In this section we work with calculating the volume of some common 3-dimensional objects.

Volume Formulas for Common 3-Dimenstional Objects			
Rectangular Solid	**Sphere**	**Right Circular Cylinder**	**Right Circular Cone**
			
Volume: $V = l\,w\,h$	Volume: $V = \frac{4}{3}\pi r^3$	Volume: $V = \pi r^2 h$	Volume: $V = \frac{1}{3}\pi r^2 h$

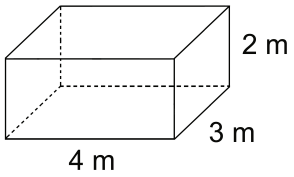
EXAMPLE 1 Find the volume of the rectangular solid.



SOLUTION: From the figure we see that the length is 10 cm, the width is 2 cm, and the height is 3 cm. So with $l = 10$ cm, $w = 2$ cm, and $h = 3$ cm the formula for the volume of a rectangular solid gives

$$V = l\,w\,h = (10\,\text{cm})(2\,\text{cm})(3\,\text{cm}) = 60\,\text{cm}^3.$$

EXAMPLE 2 Find the volume of the rectangular solid.



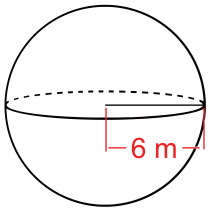
SOLUTION: From the figure we see that the length is 4 m, the width is 3 m, and the height is 2 m.

So with $l = 4$ m, $w = 3$ m, and $h = 2$ m, the formula for the volume of a rectangular solid gives

$$V = lwh$$

$$= (4\,\text{m})(3\,\text{m})(2\,\text{m}) = 24\,\text{m}^3.$$

EXAMPLE 3 Find the volume of the sphere. Approximate the value of π with 3.14.



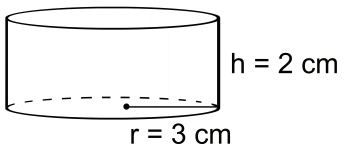
SOLUTION: From the figure we see that the radius of the sphere is 6 m.

With $r = 6$ m, the formula for the volume of a sphere gives

$$V = \frac{4}{3}\pi r^3 \approx \frac{4}{3}(3.14)(6\,\text{m})^3$$

$$= 904.32\,\text{m}^3$$

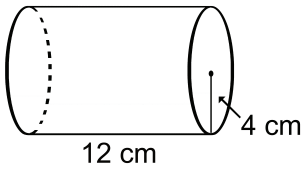
EXAMPLE 4 Find the volume of the right circular cylinder. Approximate the value of π with 3.14. Round your answer to the nearest tenth.



SOLUTION: From the figure we see that the radius of the cylinder is 3 cm and the perpendicular height is 2 cm. So with $r = 3$ cm and $h = 2$ cm, the formula for the volume of a right circular cylinder gives

$$V = \pi r^2 h \approx 3.14(3\,\text{cm})^2(2\,\text{cm}) = 56.5\,\text{cm}^3.$$

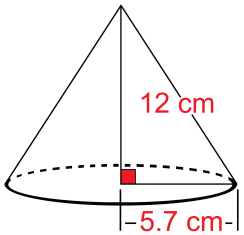
EXAMPLE 5 Find the volume of the right circular cylinder. Approximate the value of π with 3.14. Round your answer to the nearest tenth.



SOLUTION: From the figure we see that the radius of the cylinder is 4 cm and the perpendicular height is 12 cm. So with $r = 4$ cm and $h = 12$ cm, the formula for the volume of a right circular cylinder gives

$$V = \pi r^2 h \approx 3.14(4\,\text{cm})^2(12\,\text{cm}) = 602.9\,\text{cm}^3.$$

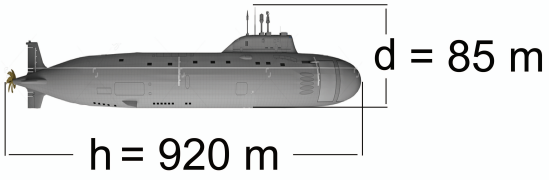
EXAMPLE 6 Find the volume of the right circular cone. Approximate the value of π with 3.14. Round your answer to the nearest hundredth.



SOLUTION: From the figure we see that the radius of the base of the cone is 5.7 cm and the perpendicular height of the cone is 12 cm. So with $r = 5.7$ cm and $h = 12$ cm, the formula for the volume of a right circular cone gives

$$V = \frac{1}{3}\pi r^2 h \approx \frac{1}{3}(3.14)(5.7\,\text{cm})^2(12\,\text{cm}) = 408.07\,\text{cm}^3.$$

EXAMPLE 7 We can approximate the volume of a submarine by assuming it is a right circular cylinder. Approximate the volume of a submarine that has a diameter of 85 m and a height of 920 m. Approximate the value of π with 3.14. Round your answer to the nearest whole number.



SOLUTION: With $r = 920$ m and $h = 85$ m, the formula for the volume of a right circular cylinder gives

$$V = \pi r^2 h \approx 3.14(42.5\,\text{m})^2(920\,\text{m}) = 5,217,895\,\text{m}^3.$$

Section 7.4 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1- 3, answer in your own words.

1) If the dimensions of a figure are given in feet, what would the units of its area be?

2) If the dimensions of a figure are given in meters, what would the units of its volume be?

3) What is the formula for the volume of a rectangular solid?

For problems 4 - 7, find the volume of each rectangular solid. Use 3.14 **to approximate** π **and round your answer to the nearest hundredth.**

4) A cube with a side of 2 cm.

5) A rectangular solid with length 3 inches, width 5 inches and height 7 inches.

6) A rectangular solid with length 2.5 cm, width 0.65 cm and height 1.8 cm.

7) A rectangular solid with length $\frac{1}{3}$ inches, width $\frac{2}{5}$ inches and height $\frac{3}{4}$ inches.

8) What is the formula for the volume of a sphere?

9) What is the formula for the volume of a right circular cone?

10) What is the formula for the volume of a right circular cylinder?

For problems 11 - 21, find the volume of each solid. Use 3.14 **to approximate** *pi* **and round your final answers to the nearest tenth.**

11) A sphere with a radius of 7 cm.

12) A sphere with a diameter of 10 inches.

13) A right circular cylinder with a radius of 8 inches and a height of 10 inches.

14) A right circular cylinder with a diameter of 3 feet and a height of 2 feet.

15) A right circular cone with a radius of 7 cm and a height of 20 cm.

16) A right circular cone with a diameter of 40 inches and a height of 50 inches.

17) The radius of the baseball is 9 inches. Find its volume.

18) An Olympic-sized swimming pool has a length of 50 meters and a width of 25 meters. It has a constant depth of 2 meters. Find its volume.

19) A NHL hockey puck has a diameter of 3 inches and a thickness of 1 inch. Find its volume.

20) A ice cream cone has a radius of 1 inch and a height of 5 inches. Find the volume of the cone.

21) A regulation NBA basketball has a diameter of 9.5 inches. Find its volume.

Mixed Practice. If required, use $\frac{22}{7}$ to approximate π and round your answers to the nearest tenth.

22) Find the circumference of the earth (with a radius of $6,371$ km).

23) If a circle has a radius of 20 mm, what is its area?

24) Determine the perimeter of the Olympic pool with length 50 meters and width 25 meters.

25) If the two legs of a right triangle are 12 inches and 16 inches, find the length of the hypotenuse.

Review. Write a mathematical expression to represent each statement.

26) The sum of 5 and -8.

27) The difference of 12 and 8.

28) -32 decreased by 17.

29) 5 less than 27.

30) The product of -23 and -4.

Use the Order of Operations to simplify each expression.

31) $12+7\times 2$

32) $6+20\div 2$

33) $(3+10)\times 4-6$

34) $35-5\times 4$

35) $7+3^2\times 2$

Chapter 7
Section 7.4 - Volume

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

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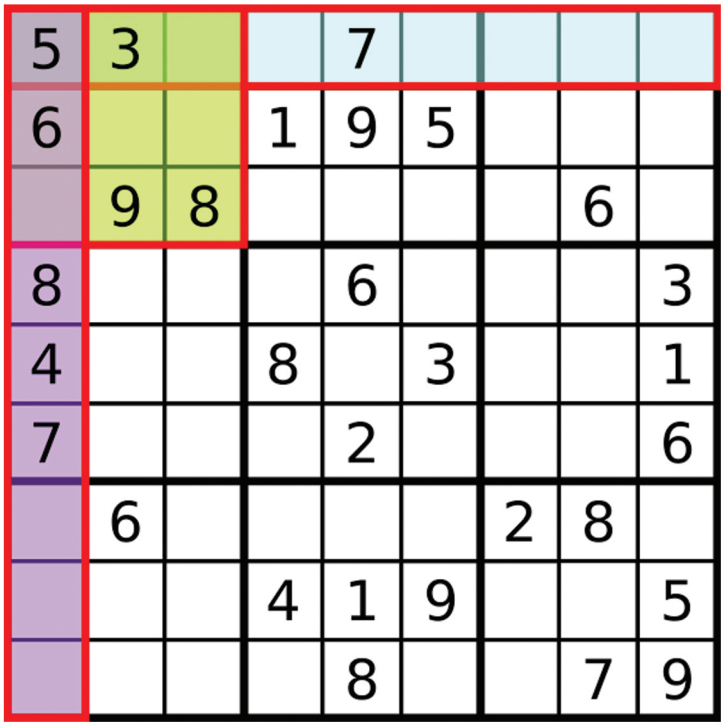
INTRODUCTION TO ALGEBRA

CHAPTER

8

Solving Sudoku



5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

Sudoku puzzles involve a lot of mathematics. Of course, the puzzles are filled with numbers, but the solution progress would be the same regardless of the symbols used. More interesting is the logic behind the solution process, which can provide extra satisfaction upon solving a puzzle (with a lot less erasing). In addition, the puzzles are examples of Latin squares-important in abstract algebra and in statistics, in experimental design.

Two Sudoku counting problems are: What is the fewest number of filled -in squares possible for a puzzle, and how many different puzzles are there? There are Sudoku puzzles with 17 numbers that have only one solution, but no one knows if there are puzzles with only 16 numbers that have a unique solution. As for the second question, there are more than five billion different puzzles. For counting purposes, puzzles that can be transformed by processes such as interchanging numbers no the top two rows are not considered different. This result depends on group theory and symmetry, crucial for much of modern physics and chemistry.

CHAPTER OBJECTIVES

8.1 Algebraic Expressions and Properties of Real Numbers

8.2 Algebraic Expressions and Terms

8.3 Equations and the Addition and Subtraction Property of Equality

8.4 Equations and the Multiplication and Division Property of Equality

8.5 Solving General Linear Equations

8.6 Applications of Linear Equations

8.1 Algebraic Expressions and Properties of Real Numbers

Constants and Variables

To this point we have worked primarily with expressions involving numbers. We now begin to investigate **algebraic expressions**. An algebraic expression is a mathematical statement formed by combining **constants** and **variables** using the operations of addition, subtraction, multiplication, division, and raising to powers.

A "constant" is a value that does not change it remains constant. A "variable" is an unknown value typically denoted with lower case letters. In section 6.3 we used variables to represent an unknown part of a percent proportion.

\$69 is 30% of how many dollars?

SOLUTION: Consider the part of the statement "30% of how many dollars?" This implies that the *Base* is unknown. Setting up the percent proportion gives

$$\frac{Amount}{Base} = Percent \Rightarrow \frac{69}{Base} = 30\%.$$

Converting the percent to a fraction we get

$$\frac{69}{Base} = \frac{30}{100} \Rightarrow \frac{69}{Base} = \frac{3}{10}.$$

Since these are equivalent fractions their numerators and denominators must be the same. Therefore,

$$\frac{69}{Base} = \left(\frac{3}{10}\right)\left(\frac{23}{23}\right) = \frac{69}{230}.$$

This implies that the *Base* = 230 and so, \$69 is 30% of \$230.

In the above example, the *Amount*, \$69, and the *Percent*, 30%, are constants while the unknown value of the *Base* was denoted with the variable b.

The previous example illustrates the use of what's called an "algebraic equation" since the expression used to solve the problem used an equal sign. In this section we study how to set-up algebraic expressions and how we can use properties of Real Numbers to manipulate them. In subsequent sections we develop techniques that can be used to set-up and solve algebraic equations.

Writing Phrases as Algebraic Expressions

When writing algebraic expressions we will use both constants and variables. We typically use lower case letters to denote variables. Some of the more common lower case letters we will be using are x, y, or n. However, any letter, uppercase or lowercase, and really any shape, can be used as a variable.

In previous chapters we worked with translating English phrases into mathematical statements (expressions). There were some commonly used phrases we used in forming the mathematical statements. A summary of these common phrases is given here.

Commonly Used Phrases for Translating Phrases to Algebraic Expressions			
Mathematical Operation			
Addition: $+$	**Subtraction:** $-$	**Multiplication:** $\times, \cdot$	**Division:** $\div$
add	subtract	multiply	divide
sum	minus	times	quotient
plus	less than	product	divided by
more than	subtracted from	of	split up
increased by	decreased by		

The examples provided here illustrate how to translate these commonly used phrases into algebraic expressions.

EXAMPLE 1 Consider the following phrase. *Logan earned five dollars more than his friend Brooke last weekend.* Express the amount Logan earned as an algebraic expression.

SOLUTION: The first step in translating phrases into algebraic expressions is to identify what in the statement is a constant and what is a variable. A constant will be a quantity that does not change while the variable will be an unknown value/quantity.

In this statement, we know that Logan earned *five dollars more than his friend Brooke last weekend*. We do not know how much Brook earned but we do know that Logan earned five dollars more than Brooke.

Since the unknown in this statement is the amount Brooke earned last weekend, let x represent this amount. We typically write this as

$$x = \text{amount earned by Brooke last weekend.}$$

The constant will be the \$5.

Note that many times there is information provided in the statement that has no relevance at all to the problem we are being asked to solve. In this particular problem, the irrelevant or inconsequential information is that Logan and Brooke are friends. We can ignore this going forward.

We can now directly translate the phrase into an algebraic expression. That is,

$$\text{Logan earned : } \text{five dollars more than Brooke earned last weekend}$$

$$\Rightarrow 5 + x$$

EXAMPLE 2 Consider the following phrase. *Terry's age is three years more than twice her daughter's age.* Express Terry's age as an algebraic expression.

SOLUTION: In this statement we know that Terry's age *is three years more than twice her daughter's age.* Since we do not know her daughter's age, this becomes the variable. That is,

$$\text{let } d = \text{Terry's daughter's age.}$$

We can now write the phrase *three years more than twice her daughter's age* as

$$\textit{three years more than twice } d.$$

At this point we need to know that when we say "*twice* d " we mean $2d$. So at this point we have that

$$\textit{three years more than twice } d.$$

can be written as

$$\textit{three years more than } 2\ d.$$

Last, since "more than" implies addition,

$$\textit{three years more than } 2\ d.$$

becomes

$$3+2d.$$

Therefore, the phrase *Terry's age is three years more than twice her daughter's age*, can be expresses as

$$3+2d, \quad \text{where } d = \text{her daughter's age.}$$

EXAMPLE 3 Consider the following phrase. *Jennifer's cookies were divided equally between twelve students.* Express the number of cookies each student received as an algebraic expression.

SOLUTION: In this statement we know that Jennifer's cookies were divided equally between twelve students, but we do not know how many cookies Jennifer had. So,

$$\text{let } c = \text{the number of cookies Jennifer has.}$$

Because Jennifer's cookies were divided equally between twelve students, we will use the operation of division. That is, the number of cookies each student received can be expressed as $\frac{c}{12}$.

Note that we can write $\frac{c}{12}$ a couple of different ways. We could have also written this as

$$c \div 12, \quad \text{or} \quad \frac{1}{12}c.$$

Evaluating Expressions

If we know the values of the variables in an algebraic expression we can evaluate the expression by substituting, or replacing, the variables with their known values. Once the values are substituted into an expression we can simplify the expression by following the Order of Operations. We have been applying the Order of Operations throughout the book but we provided them here again for a reminder.

Order of Operations
1) Perform all operations inside parentheses, or any grouping symbols, first.
2) Simplify any expressions containing exponents.
3) Perform multiplication and division as they occur from left to right.
4) Perform addition and subtraction as they occur from left to right.

EXAMPLE 1 Evaluate the expression $2x+7$ when $x=10$.

SOLUTION: The first step in evaluating an algebraic expression is to substitute the given value of the variable into the expression.

Since we are given that $x=10$,

$$2x+7 \text{ becomes } 2(10)+7.$$

Using the Order of Operations this simplifies to

$$2(10)+7=20+7$$

$$=27.$$

EXAMPLE 2 Evaluate the expression $-2(x-y)$ when $x=1$ and $y=3$.

SOLUTION: Replacing the variables with the provided values, $x=1$ and $y=3$, we get that

$$-2(x-y) \text{ becomes } -2(1-3).$$

The Order of Operations directs us to simplify within grouping symbols first so we get that

$$-2(1-3)=-2(-2).$$

To finish we calculate the indicated product. Therefore, the answer is

$$-2(1-3)=4.$$

EXAMPLE 3 Evaluate the expression $(-5m^2+6)\div n$ when $m=4$ and $n=2$.

SOLUTION: Replacing the variables with the provided values, $m=4$ and $n=2$, we get that

$$(-5m^2+6)\div n \text{ becomes } (-5(4)^2+6)\div 2$$

Using the Order of Operations to simplify gives

$$(-5(4)^2+6)\div 2 = (-5(16)+6)\div 2$$

$$= (-80+6)\div 2$$

$$= -74\div 2$$

$$= -37.$$

Properties of Real Numbers

The set of Real Numbers follow rules, or properties, that you will find familiar. Here, the goal is to name these properties and apply them to algebraic expressions. The first property we consider is called the **Commutative Property of Real Numbers**.

Commutative Property of Real Numbers
For any real numbers x and y,
Commutative Property of Addition: $x+y=y+x$.
Commutative Property of Multiplication: $xy=yx$.

If these properties don't look familiar, replace x and y with numbers. That is, we know that

$$3+4 \text{ is the same as } 4+3$$

since both sum to 7. This is what the Commutative Property of Addition says.

Moreover, we know that

$$5(8) \text{ is the same as } 8(5)$$

since both multiply to 40. This is what the Commutative Property of Multiplication says.

Since variables represent unknown Real Numbers, these properties apply to variables as well.

EXAMPLE 1 Use the Commutative Property of Real Numbers the write an equivalent expression for $y+7$.

SOLUTION: According to the Commutative Property of Addition, we can either add 7 to y, or we can add y to 7. Both provide the same result. Therefore,

$$y+7 \text{ is equivalent to } 7+y$$

$$\Rightarrow y+7=7+y.$$

EXAMPLE 2 Use the Commutative Property of Real Numbers the write an equivalent expression for $-2+x$.

SOLUTION: According to the Commutative Property of Addition, we can either add x to -2, or we can add -2 to x. Both provide the same result. Therefore,

$$-2+x \text{ is equivalent to } x+(-2)$$

$$\Rightarrow -2+x=x+(-2) \text{ or } -2+x=x-2.$$

EXAMPLE 3 Use the Commutative Property of Real Numbers the write an equivalent expression for ab.

SOLUTION: The Commutative Property of Multiplication says we can either multiply b by a, or we can multiply b by a. They both result in the same product. Therefore,

$$ab \text{ is equivalent to } ba \Rightarrow ab=ba.$$

Helpful Hint: Commutative Property of Real Numbers

The Commutative Property of Real Numbers says that we can add and multiply real numbers in any order we choose. This does not apply to subtraction and division.

While $3+4=4+3$, it is not true that $3-4=4-3$ since

$$3-4=-1 \text{ while } 4-3=1, \text{ and } -1\neq 1.$$

However, if we write subtraction in terms of addition we can apply the Commutative Property of Addition. Therefore, the following **is** true.

$$8-6=8+(-6)=-6+8 \text{ since } 8-6=2=-6+8.$$

Division works the same way. There is no Commutative Property of Division. This can be seen with the simple example that

$$\frac{8}{2}\neq\frac{2}{8}.$$

However, if we write division in terms of multiplication then we can apply the Commutative Property of Multiplication. That is,

$$\frac{8}{2}=8\left(\frac{1}{2}\right)=\frac{1}{2}(8).$$

The next property we study is called the **Associative Property of Real Numbers**. It says that when we are adding or multiplying more than two numbers at a time, we can group the numbers together in any order we wish. For example,

$$5+8+2=(5+8)+2=13+2=15 \text{ or } 5+8+2=5+(8+2)=5+10=15.$$

It does not matter if we add 5 with 8 and then add 2, or if we add 8 and 2 and then add 5.

This same idea applies to multiplication. As an example,

$$3(-2)(5) = [3(-2)](5) = -6(5) = -30 \quad \text{or} \quad 3(-2)(5) = 3[-2(5)] = 3(-10) = -30.$$

Associative Property of Real Numbers
For any real numbers x, y, and z
Associative Property of Addition: $x+(y+z) = (x+y)+z$.
Associative Property of Multiplication: $x(yz) = (xy)z$.

EXAMPLE 4 Use the Associative Property of Real Numbers the write an equivalent expression for $-2+(x+y)$.

SOLUTION: According to the Associative Property of Multiplication, we can either add x and y and then add -2, or we can -2 and x and then add y. Therefore,

$$-2+(x+y) \text{ is equivalent to } (-2+x)+y$$

$$\Rightarrow -2+(x+y) = (-2+x)+y.$$

EXAMPLE 5 Use the Associative Property of Real Numbers the write an equivalent expression for $(a-b)+c$.

SOLUTION: First note that we should rewrite the subtraction in terms of addition. That is,

$$(a-b)+c = (a+(-b))+c.$$

Applying the Associative Property of Addition then gives that

$$(a+(-b))+c \text{ is equivalent to } a+(-b+c)$$

$$\Rightarrow (a-b)+c = a+(-b+c).$$

EXAMPLE 6 Use the Associative Property of Real Numbers the write an equivalent expression for $5(xy)$.

SOLUTION: According to the Associative Property of Multiplication we can either multiply x and y and then multiply by 5, or we can multiply 5 and x and then multiply by y. Therefore,

$$5(xy) \text{ is equivalent to } (5x)y$$

$$\Rightarrow 5(xy) = (5x)y.$$

The next property of Real Numbers we look at it is called the **Distributive Property for Real Numbers**. This property deals with multiplying a sum or difference by another number. As an example of what the Distributive Property for Real Numbers says, consider the following.

$$3(5+7) = 3(12) = 36, \text{ however, } 3(5+7) = 3(5)+3(7) = 15+21 = 36.$$

In the second expression we "distributed" the 3 over the sum $5+7$.

Because we can write subtraction in terms of addition, the Distributive Property for Real Numbers applies to subtraction as well. That is,

$$2(8-6) = 2(8+(-6)) = 2(8)+2(-6) = 16+(-12) = 4.$$

Distributive Property of Real Numbers
For any real numbers x, y, and z
Multiplication over Addition: $x(y+z) = xy+xz$.
Multiplication over Subtraction: $x(y-z) = xy-xz$.

The next few examples illustrate how the Distributive Property is used.

EXAMPLE 7 Use the Distributive Property to clear the parentheses in the expression $-2(3x-5)$.

SOLUTION: It is sometimes easier to keep track of signs by writing any subtraction as addition. Therefore, rewrite

$$-2(3x-5) \text{ as } -2(3x+(-5))$$

and apply the Distributive Property.

Therefore,

$$-2(3x+(-5)) = -2(3x)+(-2)(-5).$$

Using the Associative Property of Real Numbers on this result we that

$$\begin{aligned} -2(3x-5) &= -2(3x+(-5)) \\ &= -2(3x)+(-2)(-5). \\ &= (-2(3))x+10 \\ &= -6x+10. \end{aligned}$$

EXAMPLE 8 Use the Distributive Property to clear the parentheses in the expression $-(7x-8)$.

SOLUTION: Recall from section 2.3 that we defined the opposite of the number a as $-1(a)$, or that $-a = -1(a)$. This means that

$$-(7x-8) \text{ is really } -1(7x-8).$$

Moreover, if we write the subtraction, $7x-8$, in terms of addition and apply the Distributive Property we get

$$-(7x-8) = -1(7x+(-8))$$

$$= -1(7x)+(-1)(-8)$$

$$= (-1(7))x+8$$

$$= -7x+8.$$

EXAMPLE 9 Use the Distributive Property to clear the parentheses in the expression $\frac{1}{3}(3x+6y-9z)$.

SOLUTION: Applying the techniques from the previous examples we see that

$$\frac{1}{3}(3x+6y-9z) = \frac{1}{3}(3x+6y+(-9z))$$

$$= \frac{1}{3}(3x)+\frac{1}{3}(6y)+\frac{1}{3}(-9z)$$

$$= \left(\frac{1}{3}(3)\right)x+\left(\frac{1}{3}(6)\right)y+\left(\frac{1}{3}(-9)\right)z$$

$$= \frac{3}{3}x+\frac{6}{3}y+\frac{-9}{3}z$$

$$x+2y-3z.$$

Section 8.1 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 4, write the indicated quantity as an algebraic expression. Clearly define your variable.

1) *Leo is* 2 *years older than his brother*. Express Leo's age as an algebraic expression.

2) *Rachel drives twice as fast as her mother*. Express Rachel's speed as an algebraic expression.

3) *A bank account (b) is decreased by* \$34.. Express the amount of money in the account as an algebraic expression.

4) *A pizza is split up evenly between* 8 *people.*. Express the amount of pizza each person receives as an algebraic expression.

For problems 5 - 9, evaluate the expression for the given values of the variables.

5) $5x-4y$ for $x=-2$ and $y=7$ 6) $-4a$ for $a=-6$ 7) b^2+7 for $b=-4$

8) $5(x-y)$ for $x=3$ and $y=-2$ 9) $-2a^2$ for $a=-5$

For problems 10 - 14, write an equivalent expression using the Commutative Property of Real Numbers.

10) $x+5$ 11) $-9+b$ 12) ab 13) $x(3)$

14) $10-c$

For problems 15 - 19, write an equivalent expression using the Associative Property of Real Numbers.

15) $-4+(x+3)$ 16) $5(6x)$ 17) $(x-2)+7$ 18) $a(bc)$

19) $3+(x+y)$

For problems 20 - 24, write an equivalent expression using the Distributive Property of Real Numbers.

20) $-4(x+3)$ 21) $8(a-3b+c)$ 22) $-6(-x+2y)$

23) $-(2x-4)$ 24) $\frac{1}{2}(8x-10y-4)$

Mixed Exercises.

25) Evaluate $-3x^2$ for $x=2$. 26) Evaluate $5a+4$ for $a=-7$

27) Evaluate $8c-4d$ for $c=-1$ and $d=9$.

28) Clear the parentheses for $-2(x+3)$

29) Rewrite $6+(a+7)$ using the Associative Property of Real Numbers.

Chapter 8
Section 8.1 – Algebraic Expressions and Properties of Real Numbers

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

1) _____

2) _____

3) _____

4) _____

5) _____

6) _____

7) _____

8) _____

9) _____

10) _____

11) _____

12) _____

13) _____

14) _____

15) _____

16) _____

17) _____

18) _____

19) _____

20) _____

21) _____

22) _____

23) _____

24) _____

25) _____

26) _____

27) _____

28) _____

29) _____

8.2 Algebraic Expressions and Terms

Terms

In the previous section we defined an algebraic expression to be a mathematical statement formed by combining constants and variables using the operations of addition, subtraction, multiplication, division, and raising to powers. A **term** within an algebraic expression is itself an expression. However, a term is an expression formed by combining constants and variables using the operations of multiplication, division, and raising to powers. Note that when forming a term, the operations of addition and subtraction are not used. This means that terms within an algebraic expression are separated by addition or by subtraction.

For an example, in the expression

$$5x+7z,$$

the expressions $5x$ and $7z$ are separated by addition. This means that the terms of the expression are

$$5x \text{ and } 7z.$$

Because we will want to know the signs of the terms within an algebraic expression, it is advisable to write subtraction using addition. As an example, the terms in the expression

$$3x-8$$

are not $3x$ and 8. Because we want to recognize all of the information contained within each term, we need to record the sign of each term as well. That is, since

$$3x-8=3x+(-8),$$

the terms of the algebraic expression are $3x$ and -8.

Terms and Coefficients

Again, terms are expressions formed by combining constants and variables using the operations of multiplication, division, and raising to powers. The constant portion of the term is called its **coefficient**. That is, in the expression

$$3x - 8y + z,$$

the terms are $3x$, $-8y$, and z. Each of these terms has two parts, the constant part and the variable part. Since the constant part of the term $3x$ is 3, we say that the "coefficient of the term $3x$ is 3." Likewise, the coefficient of the term $-8y$ is -8. Note that the coefficient of the term z is 1. This is because $z = 1(z)$.

Avoiding Mistakes

Variables written without a coefficient are understood to have a coefficient of 1. That is, the term xy is understood to have a coefficient of 1: $xy = 1xy$.

We also classify terms in two different ways. There are *variable terms* and *constant terms*. Variable terms are terms that contain a variable and constant terms are terms that do not contain a variable. In the above expression $3x - 8y + z$, every term contains a variable so each term is classified as *variable*. In the expression $3x - 8$, the term -8 would be classified as a *constant* term because it does not contain a variable.

EXAMPLE 1 For the expression $7a - 5b + 0.5c$, list the terms, their coefficients, and whether the term is variable or constant.

SOLUTION: First note that

$$7a - 5b + 0.5c = 7a + (-5b) + 0.5c$$

so the terms are $7a$, $-5b$, and $0.5c$.

Terms	Coefficient	Classification
$7a$	7	Variable
$-5b$	-5	Variable
$0.5c$	0.5	Variable

EXAMPLE 2 For the expression $-2x-y+7$, list the terms, their coefficients, and whether the term is variable or constant.

SOLUTION: First note that

$$-2x-y+7=-2x+(-1y)+7$$

so the terms are $-2x$, $-y$, and 7.

Terms	Coefficient	Classification
$-2x$	-2	Variable
$-y$	-1	Variable
7	7	Constant

Combining Like Terms

Like Terms are terms that share the same variables raised to the same powers. For example,

$$5x \text{ and } 7x$$

are considered to be like terms since the variable portion of each terms is x. On the other hand,

$$-3yz^2 \text{ and } 2y^2z^2$$

are not like terms became the variable portion of $-3yz^2$, yz^2, is not the same as the variable portion of $2y^2z^2$, y^2z^2. Both terms contain the same variables, y and z, and both terms contain z raised to the second power but one term contains y^1 while the other contains y^2. So even though the terms contain the same variables, the variables are not raised to the same powers.

Combining Like Terms refers to adding/subtracting like terms. There is a general rule to follow when combining like terms.

Combining Like Terms

To combine like terms, add (or subtract) the coefficients of the terms. The variable portion of the resulting term is the same as the variable portion of the original terms.

EXAMPLE 1 Simplify the expression: $7x - 12x$.

SOLUTION: When told to "simplify" an expression, the goal is to combine like terms. In this expression the two terms are

$$7x \text{ and } -12x.$$

The variable portion of each term is x so they are like terms and we can combine them. To combine like terms we add (or subtract) their coefficients. The variable portion of the result is the same as the variable portion of the original two terms. Therefore,

$$7x - 12x = 7x + (-12)x$$

$$= (7 + (-12))x$$

$$= (-5)x$$

$$= -5x.$$

EXAMPLE 2 Simplify the expression: $-3x^2 + 4x - 5x^2 - x$.

SOLUTION: In this expression there are four terms.

$$-3x^2,\ 4x,\ -5x^2,\ \text{and}\ -x.$$

Like terms are terms in which the variable portions are the same, so in this expression, the like terms are

$$-3x^2 \text{ and } -5x^2, \text{ and } 4x \text{ and } -x.$$

Because addition is commutative we can rearrange the terms in the expression as follows.

$$-3x^2+4x+(-5x^2)+(-1x)$$

$$=-3x^2+(-5x^2)+4x+(-1x).$$

Combining like terms we then get

$$=-3x^2+(-5x^2)+4x+(-1x)$$

$$=(-3+(-5))x^2+(4+(-1))x$$

$$=(-8)x^2+(3)x$$

Therefore, $-3x^2+4x-5x^2-x$ simplifies to

$$-8x^2+3x.$$

EXAMPLE 3 Simplify the expression $\frac{1}{3}x+9-\frac{1}{5}x-12$.

SOLUTION: In this expression the like terms are

$$\frac{1}{3}x \text{ and } -\frac{1}{5}x, \text{ and } 9 \text{ and } -12.$$

Again using the fact that addition is commutative we can rearrange the expression and then combine like terms.

$$\frac{1}{3}x+9+\left(-\frac{1}{5}x\right)+(-12)$$

$$=\frac{1}{3}x+\left(-\frac{1}{5}x\right)+9+(-12)$$

$$=\left(\frac{1}{3}+\left(-\frac{1}{5}\right)\right)x+(9+(-12))$$

$$=\left(\frac{5-3}{15}\right)x+(-3)$$

$$=\left(\frac{2}{15}\right)x+(-3).$$

Therefore, $\frac{1}{3}x+9-\frac{1}{5}x-12$ simplifies to

$$\frac{2}{15}x-3.$$

Distributive Property and Combining Like Terms

Recall the Order of Operations.

Order of Operations
1) Perform all operations inside parentheses, or any grouping symbols, first.
2) Simplify any expressions containing exponents.
3) Perform multiplication and division as they occur from left to right.
4) Perform addition and subtraction as they occur from left to right.

In the context of simplifying expressions, the Order of Operations says that if we have an expression with grouping symbols, we have to simplify within the grouping symbols first. With expressions this means we will need to apply the Distributive Property of Real Numbers.

EXAMPLE 4 Simplify the expression $3(x-5)-6(2x-1)$.

SOLUTION: The first step we need to perform here is simplifying within the grouping symbols. Here, the expressions in the grouping symbols (parentheses) are

$$x-5 \text{ and } 2x-1.$$

Since neither of the terms within the parentheses are like terms, no simplification can be done.

The second step is to perform the indicated multiplication using the Distributive Property of Real Numbers. That is,

$$3(x+(-5))+(-6)(2x+(-1))$$

$$=3(x)+3(-5)+(-6)(2x)+(-6)(-1)$$

$$=3x+(-15)+(-12x)+6$$

$$=3x-15-12x+6.$$

Now we can finish simplifying by combining like terms. So,

$$3x-15-12x+6$$

$$=3x+(-12x)+(-15)+6$$

$$=(3+(-12))x+(-15+6)$$

$$=(-9)x+(-9)$$

$$=-9x-9.$$

EXAMPLE 5 Simplify the expression $-12-2(x-3y-4z)+7(2x-5y-1)-6z$.

SOLUTION: Within the parentheses there are no like terms so we can move to applying the Distributive Property to clear parentheses.

$$-12-2(x-3y-4z)+7(2x-5y-1)-6z$$

$$=-12+(-2)(x+(-3y)+(-4z))+7(2x+(-5y)+(-1))+(-6z)$$

$$=-12+(-2)(x)+(-2)(-3y)+(-2)(-4z)+7(2x)+7(-5y)+7(-1)+(-6z)$$

$$=-12+(-2x)+(6y)+(8z)+(14x)+(-35y)+(-7)+(-6z)$$

Now that the parentheses have been cleared we can continue simplifying by combining like terms.

$$-12+(-2x)+(6y)+(8z)+(14x)+(-35y)+(-7)+(-6z)$$

$$=(-2x)+(14x)+(6y)+(-35y)+(8z)+(-6z)+(-12)+(-7)$$

$$=(-2+14)x+(6+(-35))y+(8+(-6))z+(-12)+(-7)$$

$$=(12)x+(-29)y+(2)z+(-19)$$

Therefore,

$$-12-2(x-3y-4z)+7(2x-5y-1)-6z$$

simplifies to

$$12x-29y+2z-19.$$

Section 8.2 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 4, list each term and the coefficient for each term.

1) $5x+3y$ 2) $2a-b$ 3) $-8x+2y-10$ 4) $a-c+1$

For problems 5 - 10, determine if the terms are like terms or unlike terms.

5) $3x,\ 2y$ 6) $-7a,\ 10a$ 7) $-2,\ 9$ 8) $9x,\ 13x^2$

9) $-10xy,\ 9yx$ 10) $6,\ 6x$

For problems 11 - 14, identify which terms are variable terms and which terms are constant term.

11) $5x+3$ 12) $xy-8x$ 13) $-10-a$ 14) $5x^2+3x-17$

For problems 15 - 25, simplify the given expression.

15) $8x-10y+2x-y$ 16) $-5ab-4ab$ 17) $-4x+12x+11y$

18) $6b-10b+18b-b$ 19) $-3x-9x^2+2x-7x^2$ 20) $x^2-10x+3x-30$

21) $5x^2+5xy+2xy-10y^2$ 22) $\frac{1}{5}x-6+2x$ 23) $\frac{2}{3}x+8y+12y-\frac{1}{4}x$

24) $8-12x+10-9x$ 25) $6\cdot 5a+7\cdot 2b-10\cdot 1\,a-13\cdot 5b$

For problems 26 - 35, simplify the given expression.

26) $5(x-4)+2(3x+1)$ 27) $10-3(x-4)$ 28) $-7-(3a+10)$

29) $8(y-6)-7(5y+6)$ 30) $-4(6x-4)-7(8x+1)$ 31) $8-2(12a-5)$

32) $5(3x-2y)-10(x+9y)$ 33) $-(b+9)-(b+1)$

34) $14+7(x-5)+3(3x+1)-10x$

35) $3(x+2y-3z)-4(3x+y+2z)$

Chapter 8
Section 8.2 – Algebraic Expressions and Terms

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

1)

Term		
Coefficient		

2)

Term		
Coefficient		

3)

Term			
Coefficient			

4)

Term			
Coefficient			

11)

Term			
Classification			

12)

Term			
Classification			

13)

Term			
Classification			

14)

Term			
Classification			

5) _____

6) _____

7) _____

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35) _____

8.3 Equations and the Addition and Subtraction Property of Equality

Equations and Solutions to Equations

In the previous sections we defined an algebraic expression to be a mathematical statement formed by combining constants and variables using the operations of addition, subtraction, multiplication, division, and raising to powers. An **algebraic equation** is an expression that *equates* two algebraic expressions. Another way to say this is that an algebraic equation is a mathematical statement that contains an equal sign between two algebraic expressions. For example,

$$3 = -5 + 2(1 - x), \quad 4z - 5 = 2(z + 5), \quad \text{and} \quad 5y^2 + 7 = 0$$

are all examples of equations.

In this course we focus on solving what are called **linear equations in one variable**. The equations we consider will only contain one variable and they are called "linear" because the highest power on the variable is one. To "solve" an equation means to find a value of the variable that makes the equation true. Notice that of the examples above, the only equation that is not a linear equation in one variable is

$$5y^2 + 7 = 0.$$

The next few examples demonstrate what a solution to an equation is and how to check whether or not it makes the equation true.

EXAMPLE 1 Determine if $x = 4$ is a solution to the equation $2x - 3 = 5$.

SOLUTION: If the solution to the equation is $x = 4$, then it should make the equation true. What this means is that if we substitute 4 in for x and simplify both sides of the equation we should end up with a true statement.

Substituting $x = 4$ gives that

$$2x - 3 = 5 \text{ becomes } 2(4) - 3 = 5.$$

To determine whether or not this is a true statement we need to simplify. That is,

$$2(4) - 3 = 5$$

$$\Rightarrow 8 - 3 = 5$$

$$\Rightarrow 5 = 5. \checkmark$$

Since $5 = 5$ is a true statement, $x = 4$ is the solution to $2x - 3 = 5$.

EXAMPLE 2 Determine if $x = -\frac{1}{5}$ is a solution to the equation $9 = -15x + 7$.

SOLUTION: Substituting $x = -\frac{1}{5}$ into the given equation gives that

$$9 = -15x + 7 \text{ becomes } 9 = -15\left(-\frac{1}{5}\right) + 7.$$

To determine if $x = -\frac{1}{5}$ is the solution we need to simplify. That is,

$$9 = -15\left(-\frac{1}{5}\right) + 7$$

$$\Rightarrow 9 = 3 + 7$$

$$\Rightarrow 9 = 10.$$

Because $9 \neq 10$, it is a false statement and $x = -\frac{1}{5}$ is **not** a solution to $9 = -15x + 7$.

Addition and Subtraction Property of Equality

Ultimately we need to be able to find the solution to a linear equation in one variable. There are few properties of equality that will help us accomplish this. In this section we will practice using what is called the **Addition and Subtraction Property of Equality**. The Addition and Subtraction Property of Equality can be summed up with the following two examples.

It is true that $3 = 3$. It is also true that because $3 = 3$,

$$3+8 = 3+8, \text{ since } 11 = 11.$$

Moreover, because $3 = 3$,

$$3-8 = 3-8, \text{ since } -5 = -5.$$

The idea here is that an equation is a mathematical statement equating two algebraic expressions and if these algebraic expressions are equal, they remain equal if we add or subtract the same quantity on both "sides" of the equation.

Addition and Subtraction Property of Equality
For any real numbers a, b, and c,
If $a = b$, then $a+c = b+c$, or $a-c = b-c$.

Quick Note

The Addition and Subtraction Property of Equality states that we can add/subtract the same quantity to both sides of an equation and retain equality.

Again, the Addition and Subtraction Property of Equality says that we can add/subtract the same quantity to both sides of an equation and retain equality.

Make the observation that because subtraction can be written in terms of addition, the Addition and Subtraction Property of Equality is sometime referred to simply as the Addition Property of Equality. That is,

$$a-c = b-c \text{ can be written as } a+(-c) = b+(-c).$$

When using the Addition Property of Equality we will be utilizing **Additive Inverses** of numbers.

Additive Inverse
For any real number a:
The additive inverse of any real number a is the opposite of a. Any real number plus its additive inverse is zero. That is, $a+(-a)=0$.

The reason we are interested in additive inverses and the Addition Property of Real Numbers is that the method we will use to "solve" an equation is the method of isolating the variable in the equation on one side of the equal sign. This method is described in the next example.

Quick Note

An equation that has a finite number of solutions is called a "Conditional Equation." The equation $x-5=6$ is a conditional equation because it has only one solution, $x=11$.

EXAMPLE 1 Solve $x-5=6$.

SOLUTION: To solve this equation we will isolate the variable x on one side of the equal sign using the Addition Property of Equality. That is, consider the following.

$$x-5=6$$

$$\Rightarrow\ x+(-5)=6$$

The left-side of the equation is $x-5$ or $x+(-5)$. This means that we are adding -5 to x. We wish to *undo* the operation of addition. To undo the operation of addition we can use the Addition Property of Equality.

Since the additive inverse of -5 is 5 (the opposite of -5 is 5), and $-5+5=0$, we will use the Addition Property of Equality and add 5 to both sides of the equation. That is,

$$x+(-5)=6$$

$$\Rightarrow\ x+(-5)+5=6+5$$

$$\Rightarrow\ x+0=11.$$

Now, since the sum of zero and any number is just that number we know that $x+0$ is just x. So,

$$x+0=11 \;\Rightarrow\; x=11.$$

Since we have "isolated" the variable x on one side of the equation the answer should be $x=11$. However, with any equation it is recommended that we check the solution and see if the answer we found actually solves the equation. We could have made a mistake in the solution resulting in the wrong answer.

So with $x=11$,

$$x-5=6 \text{ becomes } 11-5=6$$

and since $11-5=6$, the answer checks out and our solution was correct.

Avoiding Mistakes

To check a solution to an equation, always substitute the solution into the original equation.

In the above example we used the fact that $x+0$ is just x. In fact, 0 is called the **Additive Identity** for this reason. The sum of any number and zero is just that number.

EXAMPLE 2 Solve $-17=3+y$.

SOLUTION: To solve this equation we will isolate the variable on one side of the equal sign. The right-side of the equation is $3+y$. So we are adding 3 to the variable y.

To *undo* the operation of addition we can use the Addition Property of Equality.

Since the additive inverse of 3 is -3, the Addition Property of Equality states that we can add -3 to both sides of the equation and still retain equality. That is,

$$-17=3+y$$

$$\Rightarrow\; -17+(-3)=3+y+(-3)$$

Combining like terms on both sides of the equation we get

$$-20=0+y.$$

Since $0+y$ is just y we have that

$$-20 = y.$$

Checking our solution in the original equation we get that

$$-17 = 3+y \text{ becomes } -17 = 3+(-20)$$

and since $3+(-20) = -17$ our answer is correct.

Note that for this problem we are adding 3 to y. To undo this operation we need to add the additive inverse of 3 to both sides. This meant that

$$-17+(-3) = 3+y+(-3).$$

Because adding a negative is equivalent to subtraction we could have written this as

$$-17-3 = 3+y-3$$

and because we are now subtracting the same quantity on both sides of the equation we are using the Subtraction Property of Equality.

EXAMPLE 3 Solve $a+10 = -2$.

SOLUTION: The left-hand side of the equation is $a+10$. This means we are adding 10 to the variable a. To undo the operation of addition and isolate the variable, we can use the Addition Property of Equality and add the opposite of 10 to both sides. Since the opposite of 10 is -10 we get

$$a+10 = -2$$

$$\Rightarrow\ a+10+(-10) = -2+(-10)$$

Since addition of a negative is equivalent to subtraction we could have used the "Subtraction" Property of Equality to write

$$a-10=-2-10.$$

Either way, simplifying on both sides gives

$$a+0=-12$$

$$\Rightarrow\ a=-12.$$

Substituting $a=-12$ into the original equation gives

$$a+10=-2$$

$$\Rightarrow\ -12+10=-2$$

$$\Rightarrow\ -2=-2.\ \checkmark$$

Therefore, the answer is $a=-12$.

EXAMPLE 4 Solve $-\frac{5}{12}=q+\frac{4}{9}$.

SOLUTION: The right-hand side of the equation is

$$q+\frac{4}{9}.$$

To undo the addition of $\frac{4}{9}$ we add the opposite of $\frac{4}{9}$ to both sides. That is,

$$-\frac{5}{12}=q+\frac{4}{9}$$

Quick Note

It is important to remember that when solving linear equations, the Addition and Subtraction Property of Equality allow us to undo the operations of addition and subtraction so that we can isolate the variable on one side of the equation.

$$\Rightarrow -\frac{5}{12}+\left(-\frac{4}{9}\right)=q+\frac{4}{9}+\left(-\frac{4}{9}\right)$$

$$\Rightarrow -\frac{5\cdot 3}{12\cdot 3}-\frac{4\cdot 4}{9\cdot 4}=q+0$$

$$\Rightarrow \frac{-15-16}{36}=q$$

$$\Rightarrow -\frac{31}{36}=q.$$

Substituting $q=-\frac{31}{36}$ into the original equation gives

$$-\frac{5}{12}=-\frac{31}{36}+\frac{4}{9}$$

$$\Rightarrow -\frac{5}{12}=-\frac{31}{36}+\frac{4\cdot 4}{9\cdot 4}$$

$$\Rightarrow -\frac{5}{12}=-\frac{31}{36}+\frac{16}{36}$$

$$\Rightarrow -\frac{5}{12}=\frac{-15}{36}$$

$$\Rightarrow -\frac{5}{12}=-\frac{5\cdot \cancel{3}}{12\cdot \cancel{3}}$$

$$\Rightarrow -\frac{5}{12}=-\frac{5}{12}\ \checkmark$$

Therefore, the answer is $q=-\frac{31}{36}$.

Section 8.3 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 7, determine if the given number is a solution to the equation.

1) $3z-2=10;\ -4$
2) $-2=x-5;\ 3$
3) $2x-6=-6;\ 0$
4) $-5-a=-6;\ -1$
5) $2x+3=-11;\ -7$
6) $5y-5=10;\ 3$
7) $10x-4=-1;\ \frac{1}{2}$

For problems 8 - 14, solve the equation using the Addition Property of Equality.

8) $x-4=-6$
9) $2=x-10$
10) $z-21=11$
11) $y-1.2=3.4$
12) $-7+a=-4$
13) $-13=-4+y$
14) $z-\frac{2}{3}=\frac{1}{2}$

For problems 15 - 21, solve the equation using the Subtraction Property of Equality.

15) $x+7=-2$
16) $-4=x+10$
17) $z+2=1$
18) $y+5.7=-3.4$
19) $17+a=-5$
20) $-1=4+y$
21) $z+\frac{2}{3}=-\frac{1}{2}$

For problems 22 - 35, solve the equation.

22) $x+2=0$
23) $-5.9+y=-2.1$
24) $-10=-2+c$
25) $z-4=-10$
26) $q+4=4$
27) $x-\frac{3}{4}=-\frac{1}{3}$
28) $27=d+5$
29) $3.09+x=-2.11$
30) $p+1=-1$
31) $-12=z-11$
32) $-9=-5+y$
33) $p+6=-2$
34) $x-7=-7$
35) $d-14=-2$

Chapter 8
Section 8.3 – Equations and the Addition and Subtraction Properties of Equality

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

1) _____

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35) _____

8.4 Equations and the Multiplication and Division Property of Equality

Equations and Solutions to Equations

In the previous section we used the Addition and Subtraction Property of Equality to undo the actions of addition and subtraction in an equation so that we could isolate the variable on one side of the equation. In this section we look at using the Multiplication and Division Property of Equality to undo the actions of multiplication and division in equations.

Just as in the previous section we can look at numerical examples to illustrate the idea.

It is true that $10 = 10$. It is also true that because $10 = 10$,

$$3(10) = 3(10) \text{ since } 30 = 30.$$

Moreover, because $10 = 10$, it is true that

$$\frac{10}{5} = \frac{10}{5} \text{ since } 2 = 2.$$

Essentially, we can multiply or divide both sides of an equation by the same value and retain equality.

Multiplication and Division Property of Equality
For any real numbers a, b, and c,
If $a = b$, then $a(c) = b(c)$, and $\frac{a}{c} = \frac{b}{c}$ provided $c \neq 0$.

The next example demonstrates the relationship between multiplication and division and the use of the Multiplication and Division Property of Equality to solve equations. More specifically, the next example demonstrates that you can apply either multiplication to undo division, or you can apply division to undo multiplication. Either way, you will solve the equation correctly.

EXAMPLE 1 Solve $\frac{x}{3} = 5$.

SOLUTION: **Multiplication Property of Equality:** "Solving" the equation means we need to isolate the variable on one side of the equation. The left-hand side of the equation is

$$\frac{x}{3}$$

which means "divide x by 3."

To undo the operation of division, namely "divide by 3", we multiply by 3. According to the Multiplication Property of Equality as long as we multiply both sides of the equation by the same value we retain equality.

Therefore,

$$\frac{x}{3} = 5$$

$$\Rightarrow 3\left(\frac{x}{3}\right) = 3(5)$$

$$\Rightarrow \frac{3x}{3} = 15$$

$$\Rightarrow \frac{3}{3}(x) = 15$$

$$\Rightarrow 1(x) = 15$$

and since 1 times any number is that number, we have that

$$x = 15.$$

Division Property of Equality: An alternative approach would be to think of the equation as

$$\frac{1}{3}x = 5.$$

This then says "multiply x by $\frac{1}{3}$." To undo multiplication we use division. This means that to undo the operation of multiplication, namely "multiply by $\frac{1}{3}$," we should divide by $\frac{1}{3}$. Therefore,

$$\frac{1}{3}x = 5$$

$$\Rightarrow \frac{\frac{1}{3}x}{\frac{1}{3}} = \frac{5}{\frac{1}{3}}$$

$$\Rightarrow \frac{\frac{1}{3}}{\frac{1}{3}}(x) = 5(3)$$

$$\Rightarrow 1(x) = 15$$

$$\Rightarrow x = 15.$$

Checking our answer we see that if $x = 15$,

$$\frac{x}{3} = 5 \Rightarrow \frac{15}{3} = 5 \checkmark$$

so we solved the equation correctly.

In the above example we used two properties of Real Numbers. The first property we used is *any number divided by itself is* 1. We also used the property that *any number times* 1 *is that number*. This last property is why we call the number 1 the Multiplicative Identity.

Multiplicative Identity
1 is the Multiplicative Identity for the set of Real Numbers since for any real number a, $1(a) = a$.

In section 4.4.2 we defined the reciprocal of a real number a to be the real number $\frac{1}{a}$ provided $a \neq 0$. We used the reciprocals of numbers when explaining how to divide by fractions and in writing equivalent fractions. We state their importance as a property of real numbers here. It is similar to the Additive Inverses from the previous section. Any number a plus its additive inverse $-a$, sums to zero because zero is the additive identity. Because the multiplicative identity is 1 we have a slightly different definition for the multiplicative inverse.

Multiplicative Inverses
Given any real number $a \neq 0$, the multiplicative identity of a is the real number $\frac{1}{a}$. Any real number a times its multiplicative inverse is 1.

In the previous example we were asked to solve

$$\frac{x}{3} = 5.$$

This can be written as

$$\frac{1}{3}x = 5.$$

To "undo" the operation of "multiply x by $\frac{1}{3}$" we multiplied both sides of the equation by the multiplicative inverse of $\frac{1}{3}$. Since the multiplicative inverse is simply the reciprocal, we multiplied both sides of the equation by 3.

This idea is used in the upcoming examples as well.

EXAMPLE 2 Solve $\frac{2}{5}x = 8$.

SOLUTION: For this equation the left-hand side,

$$\frac{2}{5}x$$

says "multiply x by $\frac{2}{5}$. To undo multiplication we use division and if we divide both sides of the equation by the same value we retain equality.

Therefore,

$$\frac{2}{5}x = 8$$

$$\Rightarrow \frac{\frac{2}{5}x}{\frac{2}{5}} = \frac{8}{\frac{2}{5}}$$

$$\Rightarrow \frac{\frac{2}{5}}{\frac{2}{5}}(x) = 8\left(\frac{5}{2}\right)$$

$$\Rightarrow 1(x) = \frac{8 \cdot 5}{2}$$

$$\Rightarrow x = 20.$$

Quick Note

Remember the product of a number and its reciprocal is 1. This is why multiplying both sides of the equation in example 2 by $\frac{5}{2}$ isolates the variable.

An alternative approach to this problem would be to use the fact that to divide by a fraction we multiply by its reciprocal. Knowing that we need to undo "multiply by $\frac{2}{5}$" by dividing by $\frac{2}{5}$, and knowing that division by a fraction is accomplished by multiplying by its reciprocal, we could just multiply both sides by the reciprocal, or multiplicative inverse, of $\frac{2}{5}$.

$$\frac{2}{5}x = 8 \Rightarrow \left(\frac{5}{2}\right)\left(\frac{2}{5}x\right) = \left(\frac{5}{2}\right)(8)$$

$$\Rightarrow \frac{5 \cdot 2}{2 \cdot 5}x = \frac{5 \cdot 8}{2}$$

$$\Rightarrow 1x = 20$$

$$\Rightarrow x = 20.$$

Checking we get that with $x = 20$,

$$\frac{2}{5}x = 8 \text{ becomes } \frac{2}{5}(20) = \frac{2 \cdot 20}{5} = 8. \checkmark$$

Since dividing by a fraction is accomplished by multiplying by its reciprocal, we can always undo multiplication using multiplication by a reciprocal.

EXAMPLE 3 Solve $\frac{5}{3} = -\frac{3}{7}z$.

SOLUTION: The right-hand side of the equation,

$$-\frac{3}{7}z$$

says "multiply z by $-\frac{3}{7}$". To undo "multiply by $-\frac{3}{7}$" we divide by $-\frac{3}{7}$. Therefore,

$$\frac{5}{3} = -\frac{3}{7}z$$

$$\Rightarrow \frac{\frac{5}{3}}{-\frac{3}{7}} = \frac{-\frac{3}{7}z}{-\frac{3}{7}}$$

$$\Rightarrow \left(\frac{5}{3}\right)\left(-\frac{7}{3}\right) = 1\,(z)$$

So we get

$$-\frac{35}{9} = z \text{ or } z = -\frac{35}{9}.$$

Avoiding Mistakes

An equation obtained from cross multiplication can only be formed from a proportion (two fractions separated by an equal sign).

Checking we get that with $z = -\frac{35}{9}$,

$$\frac{5}{3} = -\frac{3}{7}z \text{ becomes } \frac{5}{3} = -\frac{3}{7}\left(-\frac{35}{9}\right) = \frac{5}{3}. \checkmark$$

Again note that right from the start we could have multiplied both sides of the equation by the reciprocal, or multiplicative inverse, of $-3/7$.

It is important to remember that when solving linear equations, the Multiplication and Division Property of Equality allows us to undo the operations of multiplication and division so that we can isolate the variable on one side of the equation.

Section 8.4 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems 1 - 7, determine if the given number is a solution to the equation.

1) $5z = -10;\ -2$
2) $-3 = \frac{x}{6};\ 18$
3) $\frac{2}{3}x = 4;\ 6$
4) $-4a = -20;\ -5$
5) $2.5x = -40;\ -16$
6) $-\frac{1}{2}y = 10;\ -20$
7) $3x = -1;\ -\frac{1}{3}$

For problems 8 - 21, solve the equation using the Multiplication and Division Property of Equality.

8) $\frac{x}{3} = -6$
9) $-2 = \frac{y}{-4}$
10) $\frac{1}{2}z = -7$
11) $-\frac{2}{3}a = \frac{1}{2}$
12) $\frac{z}{-2} = 4$
13) $\frac{3}{4} = \frac{1}{2}y$
14) $\frac{4}{5}z = -\frac{16}{25}$
15) $-4x = -44$
16) $-8 = 2x$
17) $-0.5z = -17$
18) $-12x = 6$
19) $0 = 5x$
20) $-1 = -y$
21) $18x = 9$

For problems 22 - 35, solve the equation using either the Addition and Subtraction, or the Multiplication and Division Property of Equality.

22) $x + 5 = -7$
23) $-4.2y = 8.4$
24) $-5 = 10x$
25) $\frac{5}{6}z = 30$
26) $-3 + y = 7$
27) $x - \frac{3}{5} = -\frac{2}{3}$
28) $27 = -9d$
29) $p - 8 = 0$
30) $x - 5 = 6.2$
31) $-12x = -18$
32) $-9x = \frac{18}{7}$
33) $c + 3 = -2$
34) $4t = 0$
35) $-14 = -2a$

Chapter 8
Section 8.4 – Equations and the Multiplication and Division Property of Equality

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

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8.5 Solving General Linear Equations

Section 8.3 used the Addition and Subtraction Property of Equality to solve linear equations. Section 8.4 used the Multiplication and Division Property of Equality. In this section we use both properties of equality to solve linear equations. Keep in mind, the following helpful hint when solving linear equations.

Helpful Hint: When Solving Linear Equations

We use the Addition and Subtraction Property of Equality to "undo" the operations or addition and subtraction.

We use the Multiplication and Division Property of Equality to "undo" the operations of multiplication and division.

Solving general linear equations can be accomplished in a step-by-step process.

Quick Note

After step 1 you have the option to "clear" fractions or decimals. This will be explained later in the section.

Step-by-Step Process to Solving Linear Equations

1) Use the Distributive Property to clear any grouping symbols on each side of the equation.

2) Combine Like Terms on each side of the equation.

3) Use the Addition and Subtraction Property of Equality to move all constants to one side of the equation and all variables to the other side of the equation.

4) Use the Multiplication and Division Property of Equality to isolate the variable

EXAMPLE 1 Solve $4x - 3 = 21$.

SOLUTION: The first step in the solution process is to clear any grouping symbols on each side of the equation using the Distributive Property. Since neither side of the equation contains any groupings we can move on to the next step.

The second step of the process is to combine like terms on each side of the equation. This step does not apply here so we move to step 3.

Step 3 says to move all constants to one side of the equation and all variables to the other side. For this equation, the variable only appears on the left-hand side. Because this is the only appearance of the variable we will leave it on the left-hand side. However, the left-hand side does contain a constant so we need to move it to the right-hand side.

Because we are subtracting 3 from $4x$, we use the Addition Property of Equality. That is,

$$4x - 3 = 21$$

$$\Rightarrow\ 4x - 3 + 3 = 21 + 3$$

Combining like terms we get

$$4x + 0 = 24 \text{ or simply } 4x = 24.$$

At this point notice that we have isolated $4x$ on the left-hand side. Because we want to isolate x, or $1x$, we have not finished. To finish, we undo the operation "multiply by 4" by using the Division Property of Equality. That is,

$$4x = 24$$

$$\Rightarrow\ \frac{4x}{4} = \frac{24}{4}$$

$$\Rightarrow\ 1x = 6$$

$$\Rightarrow\ x = 6.$$

Checking we see that with $x = 6$,

$$4x - 3 = 21 \text{ becomes } 4(6) - 3 = 21$$

$$\Rightarrow\ 24-3=21$$

$$\Rightarrow\ 21=21\ \checkmark$$

EXAMPLE 2 Solve $3(x+5)+2=8$.

SOLUTION: Step 1 is to clear any groupings using the Distributive Property. For this equation we need to distribute 3 over the expression $x+5$. That is,

$$3(x+5)+2=8$$

$$\Rightarrow\ 3x+3(5)+2=8$$

$$\Rightarrow\ 3x+15+2=8$$

Step 2 is to combine any like terms on each side of the equation. Here we have

$$3x+15+2=8$$

$$\Rightarrow\ 3x+17=8.$$

Step 3 is to isolate all of the constants on one side of the equation and the variable on the other side of the equation. Here again we see that the only occurrence of the variable is on the left-hand side. Because of this we will leave the variable on this side. We do need to move the constant though. Because we are adding 17 to $3x$ we can undo this using the Addition and Subtraction Property of Equality. So,

$$3x+17=8$$

$$\Rightarrow\ 3x+17-17=8-17$$

$$\Rightarrow 3x+0=-9$$

$$\Rightarrow 3x=-9.$$

The last step is to isolate the variable. We have isolated $3x$ on the left-hand side but we need to isolate x, or $1x$. Since we are multiplying the variable by 3 we can use the Multiplication and Division Property of Equality to undo it.

$$3x=-9$$

$$\Rightarrow \frac{3x}{3}=\frac{-9}{3}$$

$$\Rightarrow 1x=-3$$

$$\Rightarrow x=-3.$$

Checking, we get that with $x=-3$,

$$3(x+5)+2=8 \text{ becomes } 3(-3+5)+2=8$$

$$\Rightarrow 3(2)+2=8$$

$$\Rightarrow 6+2=8$$

$$\Rightarrow 8=8, \checkmark$$

The next example is one in which the variable appears on both sides of the equal sign.

EXAMPLE 3 Solve $-5(w+1)+2=2(w-5)$.

SOLUTION: Use the Distributive Property of clear the grouping symbols on both sides of the equation. That is, we need to distribute -5 over the expression $w+1$ and we need to distribute 2 over the expression $w-5$.

$$-5(w+1)+2=2(w-5)$$

$$\Rightarrow\ -5w+(-5)(1)+2=2w+2(-5)$$

$$\Rightarrow\ -5w-5+2=2w-10.$$

Collecting like terms we get

$$-5w-3=2w-10.$$

At this point we need to decide which side of the equation we wish to move the variable to. Because it does not matter which side we move the variable to, it is typical, or common practice, to move the variables to the left-hand side of an equation. Note that the only reason we need to do this is that there are variable terms on both sides of the equation.

Since we are moving the variable terms to the left-hand side we will need to undo the operation "adding $2w$" on the right-hand side. So,

$$-5w-3-2w=2w-10-2w$$

$$\Rightarrow\ -7w-3=-10.$$

Because the variable is solely on the left-hand side of the equation now, we are forced to move the constants to the right-hand side. This means we need to undo the operation "subtract 3" on the left-hand side.

$$-7w-3+3=-10+3$$

$$-7w=-7.$$

The last step is to isolate the variable. That is, we have isolated $-7w$, we want to isolate w. To accomplish this we undo the operation "multiply by -7" by dividing both sides by -7. This gives.

$$\frac{-7w}{-7} = \frac{-7}{-7}$$

$$\Rightarrow\ w = 1.$$

Checking we get with $w = 1$, that

$$-5(w+1)+2 = 2(w-5) \text{ becomes } -5(1+1)+2 = 2(1-5)$$

$$\Rightarrow\ -5(2)+2 = 2(-4)$$

$$\Rightarrow\ -10+2 = -8$$

$$\Rightarrow\ -8 = -8.\ \checkmark$$

Clearing Fractions and Decimals

There are two ways to approach solving a linear equation that has fractions. The first way is to simply solve the equation with the fractions in it. The second is to get rid of the fractions, or "clear the fractions", by multiplying both sides of the equation by the Least Common Denominator of all the fractions within the equation. The next two examples illustrate both techniques. It is left up to the individual student to decide which method they prefer.

EXAMPLE 1 Solve $5 = -\frac{1}{3} + \frac{4}{5}y$.

SOLUTION: The first solution method is to just leave the fractions alone and solve the equation as we did in the above examples.

$$5 = -\frac{1}{3} + \frac{4}{5}y\ :\ \text{undo the operation "adding } -\frac{1}{3}\text{"}$$

$$\Rightarrow\ 5+\frac{1}{3}=-\frac{1}{3}+\frac{4}{5}y+\frac{1}{3}$$

$$\Rightarrow\ \frac{16}{3}=0+\frac{4}{5}y:\ \text{combine like terms}$$

$$\Rightarrow\ \frac{16}{3}=\frac{4}{5}y:\ \text{undo the operation ``multiply by } \frac{4}{5}\text{''}$$

$$\Rightarrow\ \frac{\frac{16}{3}}{\frac{4}{5}}=\frac{\frac{4}{5}y}{\frac{4}{5}}$$

$$\Rightarrow\ \left(\frac{16}{3}\right)\left(\frac{5}{4}\right)=1y$$

$$\Rightarrow\ \frac{20}{3}=y.$$

An alternative approach is to eliminate the fractions from the problem by multiplying both sides of the equation by the Least Common Denominator of all the fractions in the equation. For this problem the denominators are 3 and 5 so the LCD is 15. Therefore,

$$15(5)=15\left(-\frac{1}{3}+\frac{4}{5}y\right):\ \text{use the Distributive Property on the right-hand side}$$

$$\Rightarrow\ 75=15\left(-\frac{1}{3}\right)+15\left(\frac{4}{5}y\right)$$

$$\Rightarrow\ 75=-\frac{15}{3}+\frac{15\cdot 4}{5}y$$

$$\Rightarrow\ 75=-5+12y$$

Now that the fractions have been eliminated we solve the equation as we did in the previous examples.

$$75+5=-5+12y+5$$

$$\Rightarrow\ 80=12y$$

$$\Rightarrow\ \frac{80}{12}=\frac{12y}{12}$$

$$\Rightarrow\ \frac{4\cdot 20}{4\cdot 3}=1y$$

$$\Rightarrow\ \frac{20}{3}=y.$$

Checking we see that with $y=\frac{20}{3}$, the original equation becomes

$$5=-\frac{1}{3}+\frac{4}{5}\left(\frac{20}{3}\right)$$

$$\Rightarrow\ 5=-\frac{1}{3}+\frac{4\cdot 20}{5\cdot 3}$$

$$\Rightarrow\ 5=-\frac{1}{3}+\frac{16}{3}$$

$$\Rightarrow\ 5=\frac{-1+16}{3}$$

$$\Rightarrow\ 5=\frac{15}{3}$$

$$\Rightarrow\ 5=5.\ \checkmark$$

EXAMPLE 2 Solve $1.5a - 4 = 5$.

SOLUTION: We can approach the solution to this equation in two ways. We can leave the decimals alone and work it as is or, we can eliminate the decimals first and then solve it. We begin by just leaving the decimals in the equation.

$$1.5a - 4 = 5 : \text{ undo the operation "add } -4\text{"}$$

$$\Rightarrow 1.5a - 4 + 4 = 5 + 4$$

$$\Rightarrow 1.5a + 0 = 9 : \text{ combine like terms}$$

$$\Rightarrow 1.5a = 9 : \text{ undo the operation "multiply by 1.5"}$$

$$\Rightarrow \frac{1.5a}{1.5} = \frac{9}{1.5}$$

$$\Rightarrow a = 6.$$

The other approach is to eliminate the decimals by multiplying both sides of the equation by an appropriate power of 10. Since the smallest decimal place value is tenths, we can multiply both sides of the equation by 10 to eliminate the decimals. That is,

$$1.5a - 4 = 5 : \text{ multiply both sides by 10 to eliminate the decimals}$$

$$\Rightarrow 10(1.5a - 4) = 10(5) : \text{ use the distributive property on the left-hand side}$$

$$\Rightarrow 10(1.5a) + 10(-4) = 50$$

$$\Rightarrow 15a - 40 = 50 : \text{ undo the operation "add } -40\text{"}$$

$$\Rightarrow 15a - 40 + 40 = 50 + 40 : \text{ combine like terms}$$

$$\Rightarrow 15a = 90 : \text{ undo the operation "multiply by 15"}$$

$$\Rightarrow \frac{15a}{15} = \frac{90}{15}$$

$$\Rightarrow a = 6.$$

Checking we see that with $a = 6$, the original equation becomes

$$1.5(6) - 4 = 5$$

$$\Rightarrow 9 - 4 = 5$$

$$\Rightarrow 5 = 5. \checkmark$$

Section 8.5 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

For problems $1-26$**, solve the given equation**

1) $3x-2=10$
2) $-3=\frac{x}{2}+5$
3) $-7a-4=4$
4) $-4c+2=-18$
5) $-10+2.5x=40$
6) $-\frac{1}{3}y+2=10$
7) $3=-1-x$
8) $-6+2x=x-1$
9) $8a-2=2a+10$
10) $z-7=-2+2z$
11) $5a=-6a+11$
12) $3d-9d+5=7-4d+3$
13) $4-y-2y=11y-3-3y$
14) $z-10-5z=-z+5+2z$
15) $-4(x+3)=20$
16) $-8+3(2a-3)=-7$
17) $7x-6(x-1)=7$
18) $-(3-x)-12x=6$
19) $5-2y=-3(5y-1)$
20) $-1+6(2a+5)=4a-3$
21) $18(b+2)=9(b-3)$
22) $-(x+5)=-7(x+1)$
23) $42(3y-4)=88(y-1)$
24) $-5(x-4)+10x-6=7x-10+6x$
25) $\frac{1}{2}(6z-12)=30$
26) $\frac{2}{3}x+\frac{1}{5}=2\left(\frac{5}{6}x-1\right)$

Chapter 8
Section 8.5 – Solving General Linear Equations

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

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18) _____

19) _____

20) _____

21) _____

22) _____

23) _____

24) _____

25) _____

26) _____

8.6 Applications of Linear Equations

Translating a Statement

EXAMPLE 1 If four more than a number is 23, what is the number?

SOLUTION: The first step here is to translate the statement "four more than a number is 23" into an equation we can solve.

The first step in any problem-solving situation is to identify what it is we are trying to find. Another way to put that is we need to identify what we do not know. For this problem the unknown, what we need to find, is the number that satisfies "four more than a number is 23.". Because the "number" is unknown,

$$\text{let } x = \text{ the unknown number.}$$

Now that we have represented the unknown with a variable, the statement "four more than a number is 23" can be written as

$$\text{"four more than } x \text{ is 23."}$$

We know from previous work that the phrase *more than* indicates addition. Therefore, replacing "four" with 4,

$$\text{"four more than } x \text{ is 23" becomes "} 4 + x \text{ is 23.}$$

Now, since "$4 + x$ is 23," we have the equation

$$4 + x = 23.$$

We can now solve this equation using the techniques from the last section.

$$4 + x - 4 = 23 - 4$$

$$\Rightarrow x = 19.$$

Checking we see that with $x = 19$ the equation becomes

$$19 + 4 = 23$$

$$\Rightarrow 23 = 23. \checkmark$$

Therefore, the number is 19.

EXAMPLE 2 Eleven less than a number is 14. What is the number?

SOLUTION: The first step in problem-solving is identifying the unknown, or identifying what it is we need to find. For this problem we need to find a number that satisfies "eleven less than a number is 14." Therefore,

let x = the unknown number.

We now write "eleven less than a number is 14" as

eleven less than x is 14.

Replacing "eleven" with 11 we have

11 less than x is 14.

Since the phrase *less than* indicates subtraction, we get

$$x - 11 \text{ is } 14.$$

This then becomes

$$x - 11 = 14.$$

Solving the equation we get

$$x - 11 + 11 = 14 + 11$$

$$\Rightarrow\ x = 25.$$

Checking we see that with $x = 25$, the equation becomes

$$25 - 11 = 14$$

$$\Rightarrow\ 14 = 14\ \checkmark$$

and the number is 25.

EXAMPLE 3 Two added to one third of a number is two sevenths. What is the number?

SOLUTION: The unknown for this problem is the number that satisfies the statement "two added to one third of a number is two sevenths", so

let x = the unknown number.

The statement can then be written as

$$2 \text{ add to } \frac{1}{3} \text{ of } x \text{ is } \frac{2}{7}.$$

Since *added to* implies addition and $\frac{1}{3}$ *of* x implies multiply $\frac{1}{3}$ and x we get

$$2 + \frac{1}{3}x \text{ is } \frac{2}{7}.$$

This then becomes

$$2+\frac{1}{3}x=\frac{2}{7}$$

which we can solve as follows.

$$2+\frac{1}{3}x-2=\frac{2}{7}-2$$

$$\Rightarrow \frac{1}{3}x=-\frac{12}{7}$$

$$\Rightarrow \frac{\frac{1}{3}x}{\frac{1}{3}}=\frac{-\frac{12}{7}}{\frac{1}{3}}$$

$$\Rightarrow x=\left(-\frac{12}{7}\right)(3)$$

$$\Rightarrow x=-\frac{36}{7}.$$

Checking we see that with $x=-\frac{36}{7}$ the equation becomes

$$2+\frac{1}{3}\left(-\frac{36}{7}\right)=\frac{2}{7}$$

$$\Rightarrow \frac{14}{7}-\frac{12}{7}=\frac{2}{7}$$

$$\Rightarrow \frac{2}{7}=\frac{2}{7}.\checkmark$$

and so the number is $-\frac{36}{7}$.

EXAMPLE 4 The product of 6 and a number is 2 more than the number. Find the number.

SOLUTION: Again, the unknown is a number that satisfies the statement "the product of 6 and a number is 2 more than the number" so

let x = the unknown number.

The statement "the product of 6 and a number is 2 more than the number" can now be written as

"the product of 6 and x is 2 more than x."

Since *product* indicates multiplication and *more than* indicates addition, the statement becomes

$$6x \text{ is } 2+x.$$

This then gives the equation

$$6x = 2+x$$

which we can solve for x.

$$6x - x = 2 + x - x$$

$$\Rightarrow 5x = 2$$

$$\Rightarrow \frac{5x}{5} = \frac{2}{5}$$

$$\Rightarrow x = \frac{2}{5}.$$

Checking we see that with $x = \frac{2}{5}$ our equation becomes

$$6\left(\frac{2}{5}\right) = 2 + \frac{2}{5}$$

$$\Rightarrow \frac{12}{5} = \frac{10}{5} + \frac{2}{5}$$

$$\Rightarrow \frac{12}{5} = \frac{12}{5} \checkmark$$

and so the number is $\frac{2}{5}$.

Perimeter

EXAMPLE 1 The length of a rectangular yard is 30 feet more than its width. If the perimeter of the yard is 340 feet, find the dimensions of the yard.

SOLUTION: Because we wish to find the dimensions of a rectangular yard there are two unknowns for this problem. Namely, the length and width of the rectangle. Therefore,

let l = the length, and let w = the width.

The first part of the statement is "the length of a rectangular yard is 30 feet more than its width." With our unknowns defined this can be written as

"l is 30 feet more than w.".

Since *more than* indicates addition, this becomes

"l is $w + 30$ or simply $l = w + 30$.

The second part of the statement tells us that the perimeter of the yard is 340 feet. Since the yard is a rectangle and the perimeter of a rectangle is given by

$$2l + 2w = P$$

we know that

$$2l + 2w = 340.$$

We have already determined that $l = w + 30$ so using this in $2l + 2w = 340$ gives the equation

$$2(w + 30) + 2w = 340.$$

Solving this equation for L gives

$$2w + 60 + 2w = 340$$

$$\Rightarrow\ 4w + 60 = 340$$

$$\Rightarrow\ 4w + 60 - 60 = 340 - 60$$

$$\Rightarrow\ 4w = 280$$

$$\Rightarrow\ \frac{4w}{4} = \frac{280}{4}$$

$$\Rightarrow\ w = 70.$$

Checking we see that with $w = 70$ our equation becomes

$$2(70 + 30) + 2(70) = 340$$

$$\Rightarrow 2(100)+140=340$$

$$\Rightarrow 200+140=340$$

$$\Rightarrow 340=340. \checkmark$$

Therefore, $w=70$ feet and $l=w+30=70+30=100$ feet.

Other

EXAMPLE 1 Six students in a club are awarded scholarships. If these 6 students are 25% of the total membership of the club, find the total number of students in the club.

SOLUTION: For this problem we are looking for the total number of students in a club so

let x = the total number of students in the club.

We can translate "6 students are 25% of the total membership of the club" directly as

$$6=(25\%)x \Rightarrow 6=0.25x.$$

Solving gives

$$\frac{6}{0.25}=\frac{0.25x}{0.25}$$

$$\Rightarrow 24=x.$$

Therefore, there are 24 students in the club.

EXAMPLE 2 Natasha purchased a new soccer ball and a new pair of cleats for $330. If the cleats cost $120 more than the soccer ball, how much did the soccer ball cost?

SOLUTION: This problem is similar to the Perimeter problem in that there are two unknowns. Here the two unknowns are the price of the soccer ball and the price of the cleats. Therefore,

let b = the price of the soccer ball, and let c = the price of the cleats.

We know that the total price of the two items was $330 so we know that

$$b + c = \$330.$$

We also know that the cleats were 120 more than the ball so

$$c = \$120 + b.$$

If we use this expression for c in the equation $b + c = \$330$ we get

$$b + (\$120 + b) = \$330.$$

Solving this for b we get

$$b + \$120 + b = \$330$$

$$\Rightarrow\ 2b + \$120 = \$330$$

$$\Rightarrow\ 2b + \$120 - \$120 = \$330 - \$120$$

$$\Rightarrow\ 2b = \$210$$

$$\Rightarrow \frac{2b}{2} = \frac{\$210}{2}$$

$$\Rightarrow b = \$105.$$

Since $c = \$120 + b$, $c = \$120 + \$105 = \$225$. Therefore, the ball cost \$105 and the cleats cost \$225.

Section 8.6 Exercises

Directions: Complete the following problems. You must show sufficient work in order to receive credit.

Translate the statement(s) into an equation and solve.

1) Five times a number is 20. Find the number.

2) The quotient of a number and 3 is 5. What is the number?

3) If eight more than a number is 14, what is the number?

4) Four less than a number is one more than the two times the number. Find the number.

5) The product of a number and 2 is 22. What is the number?

6) One-half plus a number is equal to 3. Find the number.

7) The product of 4 and a number is 3 less than the number. What is the number?

8) A 12 feet piece of wood is to be cut into two pieces. One piece must be 8 feet longer than the other. Find the length of each piece.

9) The width of a swimming pool is 5 feet less than the length. If the perimeter of the pool is 50 feet, find the dimensions.

10) During a baseball tournament, Leo and Joey had 14 combined hits. If Leo had 4 more hits than Joey, how many hits did each have?

11) Marc bought a pair of sneakers and a sweatshirt for $120. If the sweatshirt cost one-half the price of the sneakers, find the cost of each.

12) Barry spent $150 on groceries. If this amount was 20% of his paycheck, how much was he paid?

13) There are 90 people on an airplane. If only 30% of the passengers have carry on luggage, find the number of passengers that do not have carry on luggage.

14) Natalie has saved $640 in the last two months. If the amount saved in the first month is $80 less than the amount saved in the second month, find the amount saved in each month.

15) While on a surfing trip, the waves have doubled in size from the day before. If the sum of the wave heights for the last two days is 18 feet, find the wave height for each day.

16) Out of 60 DSC students, 18 own a surfboard. What is the percentage of students that own a surfboard?

Chapter 8
Section 8.6 – Applications of Linear Equations

Name: _____ Date: _____

Directions: **You must show sufficient work on a separate sheet of paper in order to receive credit.** Place your final answers in the spaces provided.

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Glossary

Understanding mathematical concepts is critical in our world today. Math is used daily by nearly everyone, from lay persons to highly degreed professionals. Situations in which math is used vary from balancing a checkbook or calculating change due from a store transaction to making blueprints for the construction of an office building. Understanding how to solve math problems becomes easier as one learns math terminology. Below is a list of many common math terms and their definitions.

Absolute Value (p.67) - Absolute Value makes a negative number positive. Positive numbers and zero are left unchanged by Absolute Value.

Acute Triangle (p.261) - A triangle containing only acute angles. An acute angle is an angle whose measure is less than 90°.

Addend (p.2) - In statements using addition, the numbers that are being added are called the addends.

Additive Inverse (p.330) - The opposite of a number, or the negative of a number. A number plus its additive inverse is zero.

Addition and Subtraction Property of Equality (p.329) - The formal name for the property of equality that allows us to add or subtract the same quantity to both sides of an equation and retain equality.

Algebraic Equation (p.327) - An algebraic equation is a mathematical statement that equates two algebraic expressions.

Algebraic Expression (p.301) - An algebraic expression is formed by combining variables and constants with the operations of addition, subtraction, multiplication, division, and raising to powers.

Angle (p.256) - An angle is created by two rays that have a common initial point.

Arc - A set of points that lie on the circumference of a circle.

Area (p.274) - The space contained within a two-dimensional shape, measured in square units.

Area of a Parallelogram - The formula for the area of a parallelogram is $A = bh$ where b is the measure of the base and h is the perpendicular height of the parallelogram.

Area of a Rectangle (p.22) - The formula for the area of a rectangle is $A = lw$ where l is the measure of the length and w is the measure of the width.

Area of a Trapezoid (p.274) - The formula for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$ where h is the measure of the perpendicular height, b_1 is the measure of the bottom base, and b_2 is the measure of the upper base.

Area of a Triangle (p.274) - The formula for the area of a triangle is $A = \frac{1}{2}bh$ where b is the measure of the base and h is the measure of the perpendicular height as measured from the base.

Associative Property of Real Numbers (p.310) - The Associative Property of Real Numbers says we can change how the addends in addition and the factors in multiplication are grouped. $a + (b + c) = (a + b) + c$, and $a(bc) = (ab)c$.

Base in an Exponential Expression (p.89) - The base in the exponential expression a^n is a. The value of a is greater than zero and not equal to 1.

Borrow (p.12) - In subtraction, to take a unit from the next larger place value in the minuend so as to make a number larger than the number to be subtracted.

Center of a Circle (p.282) - The "given point" in the definition of a circle. All points on a circle are equidistant from its center.

Circle (p.279) - A set of points that are all the same distance from a given point.

Circumference (p.283) - The distance measured around a circle. The circumference of a circle with radius r is given by $C = 2\pi r$.

Coefficient (p.317) - The number that is placed in front of a variable(s). For example, in $6x$, 6 is the coefficient.

Combining Like Terms (p.318) - To combine like terms, add their coefficients.

Common Denominator (p.150) - The Least Common Multiple of the denominators from two of more fractions or rational expressions.

Commutative Property of Real Numbers (p.307) - The Commutative Property of Real Numbers states that we can switch the addends in addition and the factors in multiplication. $a+b=b+a$, and $ab=ba$.

Complementary Angles (p.259) - Two angles whose measures sum to 90°.

Complex Fraction - A fraction that contains a fraction, or fractions, in the numerator and/or the denominator.

Complex Number - A Complex Number is any number of the form $a+bi$ where a and b are Real Numbers and $i=\sqrt{-1}$ is the Imaginary Unit.

Composite Number (p.106) - A number that has factors other than 1 and itself.

Conditional Equation (p.330) - An equation that is true for some value(s) of the variable(s) and not true for others.

Conditional Inequality - An inequality that is true for some value(s) of the variable(s) and not true for others.

Congruent - "Exactly the same." Identical in regards to size and shape.

Constant (p.301) - A number, term, or expression with no variables.

Constant Term - The term in a simplified algebraic expression or equation which contains no variable(s).

Cube - A solid with six sides where each side is a congruent square. The term "cube" also refers to the resulting value when a number is multiplied by itself three times as in "three cubed" is $3\cdot 3\cdot 3=27$.

Cube Root - The cubed root of a number a is a number b where b cubed equals a.

Decimal (p.178) - A number in the decimal system (base 10 system).

Decimal Point (p.178) - A point in a decimal that separates the place value for 10^0 and the place value for 10^{-1}.

Decimal Fraction - A fraction whose denominator is a power of ten.

Degree (p.257) - A measurement unit for angles.

Degree of a Polynomial - The degree of a polynomial is the highest degree of the terms in the polynomial. The degree of a term in a polynomial is the sum of

the exponents on the variables contained within the term.

Degree of a Term - The degree of a term in a polynomial is the sum of the exponents on the variables contained within the term. For a term with one variable, the degree is the exponent on the variable.

Denominator (p.98) - The "bottom" in a fraction.

Dependent Variable - A variable whose value is determined by the value of another variable.

Diameter (p.282) - A line segment that contains the center of a circle and has its endpoints on the same circle. Diameter also refers to the length of this segment. The Diameter of a circle is twice the length of the radius.

Difference (p.8) - The result of subtraction.

Digit (p.1) - A digit refers to the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

Dimensions - On the most basic level, this terms refers to the measurements describing the size of an object. For instance, length and width are the dimensions of a rectangle.

Distribute (p.311) - To multiply an expression throughout a sum or difference.

Dividend (p.32) - The number being divided during the operation of division.

Divisor (p.32) - The number "doing the division" during the operation of division.

Equilateral Triangle (p.260) - A triangle that has three equal angles and three congruent sides.

Evaluate - To compute.

Even Number - An even number is any Integer that can be divided by 2 with a remainder of zero.

Exponent (p.24) - In the expression a^n we call n the exponent.

Exterior Angle - In a triangle, an exterior angle is equal to the sum of the measures of its two alternate interior angles.

Factor (p.21) - As a noun, it is a number or expression which evenly divides a larger number or expression. As a verb, it means to break down a given number or expression into the product of two or more numbers or expressions.

Fraction (p.98) - A symbol which expresses part of a whole. A fraction contains a numerator and a denominator.

Fractional Equation - An equation that contains fractions, or rational expressions, on one or both sides of the equal sign.

General Form for the Equation of a Line - $Ax + By = C$ where A, B, and C are Integers with A and B not both zero.

Greatest Common Factor (p.114) - The largest factor that is common among two or more numbers or expressions.

Hypotenuse (p.263) - In a right triangle, it is the side opposite the right angle. In a right triangle, it is the longest side.

Identity - An equation which is true for all Real Numbers.

Improper Fraction (p.99) - A fraction in which the numerator is larger than the denominator.

Independent Variable - A variable in a function that be chosen freely, or arbitrarily.

Inequality (p. 65) - A mathematical statement that compares two expressions using one of the symbols $<$, $>$, $\leq$, or $\geq$.

Integer (p.63) - The set of Whole Numbers together with their negatives (opposites).

Interior Angles - Angles contained within the interior of a polygon or inside two parallel lines cut by a transversal.

Intersecting Lines - Lines that share a point in common.

Irrational Number (p.284) - A number that is not a Rational Number. An Irrational Number cannot be represented as a fraction. The decimal representation of an Irrational Number is neither repeating nor terminating.

Isosceles Triangle (p.260) - A triangle with two congruent sides subtended by two congruent angles.

Leading Coefficient - The coefficient of a polynomial's leading term.

Least Common Multiple (p.143) - The smallest number, or expression, two or more numbers, or expressions, evenly divide.

Leading Term - In a polynomial in one variable, it is the term containing the variable raised to the highest power. In a polynomial of more than one variable, it is the term containing the largest sum of its variable's exponents. In either case, it is the term of highest degree in a polynomial.

Least Common Denominator (p.140) - The Least Common Multiple of two or more denominators.

Legs of a Right Triangle (p.261) - The sides of a right triangle that form the right angle.

Like Fractions (p.128) - Fractions that have the same denominator.

Like Terms (p.318) - Terms that have the same variables raised to the same powers.

Line (p.258) - The set of all ordered-pairs satisfying $Ax + By = C$ where A, B, and C are Real Numbers with A and B not both zero.

Linear Equation (p.327) - An equation in which the highest power(s) on the variable(s) is one.

Long Division (p.35) - Long division is a process of division in arithmetic, usually used when the divisor is a large number, in which each step of the division is written out.

Minuend (p.8) - The numbers involved in subtraction are referred to as the minuend and the subtrahend. The minuend is the number that is subtracted from the subtrahend.

Mixed Number (p.100) - A number containing both an Integer and a Fraction.

Monomial - A polynomial consisting of one term. A monomial is obtained by combining constants and variables using the operations of multiplication, and raising to integer powers.

Multiples (p.138) - A number that can be divided by another number with a remainder of zero.

Multiplication (p.21) - Repeated addition.

Multiplication and Division Property of Equality (p.337) - The formal name for the property of equality that allows us to multiply or divide both sides of an equation by the same quantity and retain equality. In the case of division, the divisor must be non-zero.

Natural Number (p.23) - The infinite set of numbers $1, 2, 3, 4, \ldots$.

Negative Number (p.63) - Any number less than zero.

Nonlinear Equation - An equation that contains variables raised to powers other than one.

Non-negative - Any number that is not negative. Any number that is greater than zero.

Number Line (p.64) - The Real Number Line is a line on which every point corresponds to a Real Number.

Numerator (p.98) - The "top" in a fraction.

Obtuse Angle (p.258) - An angle with measure larger than 90° but less than 180°.

Obtuse Triangle (p.258) - A triangle that contains an obtuse angle.

Odd Number - An Integer that is not divisible by 2.

Parallel Lines (p.258) - Two lines that never intersect are parallel lines.

Percentage (p.220) - A fraction with 100 as its denominator.

Percent Proportion (p.236) - A percent proportion is formed by setting a percent equal to an equivalent fraction.

Perfect Square (p.264) - Any number that is the square of a Rational Number. 25 is a perfect square because $5^2 = 25$. Likewise, $\frac{9}{4}$ is a perfect square because $\frac{9}{4} = \left(\frac{3}{2}\right)^2$.

Perimeter (p.4) - The distance around the outside of a plane figure. The perimeter of a polygon is found by summing the lengths of all its sides.

Perpendicular Lines (p.258) - Two lines that intersect at a right angle are perpendicular lines.

Pi (p.283) - The Irrational Number Pi, denoted as π, is defined to be the ratio of a circle's circumference and its diameter.

Polynomial - An expression in algebra that consists of two or more terms.

Positive Number (p.63) - A number that is greater than zero.

Power (p.24) - The terms Power and Exponent can be used interchangeably.

Prime Number (p.106) - A number that can only be divided by 1 and itself.

Product (p.21) - The result of multiplication.

Proper Fraction (p.100) - A fraction in which the numerator is less than the denominator.

Proportion - A mathematical statement that equates to ratios.

Pythagorean Theorem (p.263) - In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. If a, b, and c denote the lengths of the right triangle's legs and hypotenuse, respectively, then $a^2 + b^2 = c^2$.

Quadrilateral (p.271) - A four-sided regular closed polygon.

Quotient (p.32) - The result of dividing two numbers or expressions.

Radical (p.264) - The symbol $\sqrt{}$ used to indicate a square root. The symbol $\sqrt[n]{}$ is used to indicate an nth root.

Radicand (p.264) - The number or expression within the radical.

Radius (p.282) - The length of the line segment joining the center of a circle with a point on the circle.

Ratio - A comparison between two values. The ratio of x to y may be written as $x : y$, x/y, or x *is to* y.

Rational Equation - An equation that contains a rational expression.

Rational Expression (p.120) - A fraction in which the denominator, or both the numerator and denominator, are polynomials.

Rational Number (p.98) - A Rational Number is any number that can be written as a fraction. These numbers include Integers, terminating decimals, and repeating decimals.

Ray (p.256) - A directed line segment.

Real Numbers (p.63) - The set of Real Numbers is the set consisting of both Rational and Irrational Numbers.

Reciprocal (p.165) - The reciprocal of a number is the multiplicative inverse of the number. The reciprocal of $\frac{a}{b}$ is the $\frac{b}{a}$.

Rectangle (p.272) - A quadrilateral in which all interior angles are right angles.

Remainder (p.35) - The amount left over after performing division.

Right Angle (p.258) - An angle whose measure is $90°$.

Right Triangle (p.258) - A triangle that contains a right angle.

Scalene Triangle (p.260) - A triangle in which none of the sides or angles are equal.

Scientific Notation - A method used for writing very large, or very small, numbers as a number greater than or equal to 1 but strictly less than 10 multiplied by a power of 10.

Set - A group of objects.

Square (p.55) - The result of multiplying a number by itself. Also, a quadrilateral with equal sides and four right angles.

Square Root (p.264) - The Square Root of the number a is the number b such that $b^2 = a$.

Subtrahend (p.8) - The numbers involved in subtraction are referred to as the minuend and the subtrahend. The minuend is the number that is subtracted from the subtrahend.

Sum (p.2) - The result of adding a set of numbers or algebraic expressions.

Supplementary Angles (p.259) - Two angles whose measures sum to 180°.

Term (p.316) - A mathematical expression obtained by combining Real Numbers and variables using the operations of multiplication, division, and raising to powers.

Triangle (p.256) - A three-sided closed figure.

Trinomial- An polynomial consisting of three terms.

Unlike Fractions (p.150) - Fractions that have different denominators.

Variable (p.301) - A symbol that represents a number.

Vertice (p.256) - The point at which two lines or two rays intersect.

Volume (p.293) - The amount which can be held, as measured in cubic units.

Whole Number (p.1) - The set of Natural Numbers and zero.

TRANSLATING ENGLISH PHRASES INTO ALGEBRAIC EXPRESSIONS

ENGLISH PHRASES	ALGEBRAIC EXPRESSIONS
Ten more than a number	$x + 10$
A number added to five	$5 + x$
A number increased by thirteen	$x + 13$
Four less than ten	$10 - 4$
Six less than a number	$x - 6$
A number decreased by seven	$x - 7$
The difference between a number and three	$x - 3$
The difference between three and a number	$3 - x$
Twice a number	$2x$
Half (of) a number	$\frac{1}{2}x$ *or* $\frac{x}{2}$
A number squared *or* the square of a number	x^2
The square of five more than a number	$(x + 5)^2$
Five more than the square of a number *or* the square of a number, increased by five	$x^2 + 5$
Ten percent of a number	$0.10x$
Ten times a number	$10x$
Quotient of a number and three	$\frac{x}{3}$
Quotient of three and a number	$\frac{3}{x}$
Five times the sum of a number and two	$5(x + 2)$
Five times the difference of a number and four	$5(x - 4)$
Five is three more than a number	$5 = x + 3$
The product of two and a number is ten	$2x = 10$
One half (of) a number is ten	$\frac{1}{2}x = 10$
Ten less than eight times a number is five more than the number	$8x - 10 = x + 5$
The sum of five times a number and ten is equal to the product of 15 and the number	$5x + 10 = 15x$
The sum of two consecutive integers	$x + (x + 1)$
The sum of two consecutive even integers	$x + (x + 2)$
The sum of two consecutive odd integers	$x + (x + 2)$
The sum of the squares of two consecutive integers	$x^2 + (x + 1)^2$
The sum of the cubes of two consecutive integers	$x^3 + (x + 1)^3$

TRANSLATING ENGLISH TERMS INTO ALGEBRAIC OPERATORS

Accumulate ..... +
Add, add up, added to, addition, additional ..... +
Alike ..... =
Also ..... +
Altogether ..... +
Amounts to ..... =
Amplify, amplified by ..... ×
And ..... +
Appreciate ..... +
Are (equal) ..... =
Area ..... ×
Balances ..... =
Bigger (than) ..... +
Coincides (with) ..... =
Combine(d) ..... +
Corresponds (to) ..... =
Cut (by) ..... −
Cut by a factor of ..... ÷
Decline, declined by ..... −
Decline(d) by a factor of ..... ÷
Decrease(d) by, decrement ..... −
Decrease(d) by a factor of ..... ÷
Deduct, deducted from ..... −
Depreciate ..... −
Difference (of) ..... −
Diminished (by) ..... −
Diminished by a factor of ..... ÷
Divide, divided by, divided into ..... ÷
Double(d) ..... ×
Dropped by ..... −
Dropped by a factor of ..... ÷
Dwindle ..... −
Equal, equals ..... =
Equivalent (to) ..... =
Exceeds ..... >
Exceeds by ..... +
Factors ..... ×
Fall ..... −
Farther ..... +
Fewer than ..... −
Find the total ..... +
Fraction ..... ÷
Fraction of ..... ×
Gain ..... × or +
Gives (a result of), giving ..... =
Go (went) up ..... +
Go (went) up by a factor of ..... ×
Greater (than) ..... +
Grew by, grow by ..... +
Halved ..... ÷
Identical to ..... =
In addition (to) ..... +
In all ..... +
In excess ..... +
Increased (by), increment ..... +
Increase(d) by a factor of ..... ×
Intensified by ..... ×
Interest on ..... ×
Is (equal) ..... =
Is greater (than) ..... >
Is less (than) ..... <
Less (than) ..... −
Larger (than) ..... +
Lengthen (by), longer (than) ..... +
Lower, lowered by ..... −
Matches ..... =
Magnified by ..... ×
Minus ..... −
More (than) ..... +
Multiple, multiply, multiplied by ×
Narrower (than) ..... −
Net ..... +
Of [usually in connection with fractions] ..... ×
Older (than) ..... +
On top of ..... +
Per ..... ÷
Percent (of) ..... ×
Perimeter ..... +
Plus ..... +
Product (of) ..... ×
Quadruple(d) ..... ×
Quarter ..... ÷
Quantity ..... (…)
Quotient (of) ..... ÷
Raise, raised by, rise ..... +
Ratio (of) ..... ÷
Reciprocal (of) ..... ÷
Reduce, reduce(d) by ..... −
Reduce(d) by a factor of ..... ÷
Represents ..... =
Result (is), results (are), results in =
Same (result) as ..... =
Smaller (than) ..... −
Smaller by a factor of ..... ÷
Shorten (by), shorter (than) ..... −
Subdivide ..... ÷
Subtract, subtracted from ..... −
Sum (of), summation ..... +
Take away, take from ..... −
Tally (up) ..... +
Thrice ..... ×
Times, times as much, times larger, times more, times older ... ×
Times less, times smaller, times younger ..... ÷
Together ..... +
Total (is) ..... =
Total of ..... +
Triple(d) ..... ×
Twice ..... ×
Volume ..... ×
Was (equal), were (equal) ..... =
Wider (than) ..... +
Will be (equal) ..... =
… Years older (than) ..... +
Yields ..... =

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Concepts and Skills

Authored by Daytona State College School of Mathematics

First Edition

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