Basic Trigonometric Identities

Basic Identities

$$\sin\theta = \frac{y}{r} = \frac{opp}{hyp} \qquad \cos\theta = \frac{x}{r} = \frac{adj}{hyp} \qquad \tan\theta = \frac{y}{x} = \frac{opp}{adj}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Memorize these!

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

Memorize at least the first one

the other two can be derived by dividing the first equation by $\sin^2\theta$ or $\cos^2\theta$

Sum and Difference Identities

$$\sin(a+b) = \sin a\cos b + \cos a\sin b$$
$$\sin(a-b) = \sin a\cos b - \cos a\sin b$$
$$\cos(a+b) = \cos a\cos b - \sin a\sin b$$
$$\cos(a-b) = \cos a\cos b + \sin a\sin b$$
$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a\tan b}$$
$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a\tan b}$$

Memorize at least sum and difference of sine and cosine

If you forget the tangent identities - *tan(a+b)* can be determined by dividing *sin(a+b)* by *cos(a+b)*. Likewise, for *tan(a-b).*

Double-Angle Identities

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\cos 2\theta = 2\cos^2\theta - 1$$
$$\cos 2\theta = 1 - 2\sin^2\theta$$
$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Memorize at least sine and cosine double angle identities

If you forget the tangent identity – *tan2θ* can be determined by dividing *sin2θ* by *cos2θ*.

Even and Odd Identities

$$\sin(-x) = -\sin x \qquad \csc(-x) = -\csc x$$
$$\cos(-x) = \cos x \qquad \sec(-x) = \sec x$$
$$\tan(-x) = -\tan x \qquad \cot(-x) = -\cot x$$

Know at least the sine, cosine, and tangent identities

Sine and tangent are odd; cosine is even.

Basic Trigonometric Identities

Power Reducing Formulas

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

Memorize at least sine and cosine power reducing identities

If you forget the tangent identity – $\tan^2\theta$ can be determined by dividing $\sin^2\theta$ by $\cos^2\theta$.

Memorize the sine, cosine, and at least one of the tangent half angle identities

Half Angle Identities

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{\sin 2\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

Co-function Identities

$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta \quad \tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta \quad \csc\left(\frac{\pi}{2}-\theta\right) = \sec\theta$$

$$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta \quad \cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta \quad \sec\left(\frac{\pi}{2}-\theta\right) = \csc\theta$$

Know at least the sine and cosine versions

Product to Sum Formulas

$$\sin a \sin b = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$$

$$\cos a \cos b = \frac{1}{2}[\cos(a-b) + \cos(a+b)]$$

$$\sin a \cos b = \frac{1}{2}[\sin(a-b) + \sin(a+b)]$$

Know how to apply Product to Sum and Sum to Product Formulas

Sum to Product Formulas

$$\sin a + \sin b = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\sin a - \sin b = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$\cos a + \cos b = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\cos a - \cos b = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$