Imaginary Numbers

The equation $x^2 + 25 = 1$ has no solution in the set of real numbers. In order to solve such equations it is necessary to expand the number system by defining an Imaginary Number ***i.***

The symbol ***i*** represents an imaginary number with the property $i = \sqrt{-1}$ and $i^2 = -1$.

Using this new number *i*

$$\sqrt{-25} = \sqrt{-1 \cdot 25} = \sqrt{-1} \cdot \sqrt{25} = i\sqrt{25} = 5i$$

$$\sqrt{-36} = \sqrt{-1 \cdot 36} = \sqrt{-1} \cdot \sqrt{36} = i\sqrt{36} = 6i$$

For any positive real number n, $\sqrt{-n} = i\sqrt{n}$.
Imaginary numbers are not real numbers, and some properties of real numbers do not apply to imaginary numbers.

One such property is the Product Rule for Radicals, $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ only applies if $\sqrt[n]{a}$ and the $\sqrt[n]{b}$ exist among the set of real numbers.

The product rule does not apply to $\sqrt{-25} \cdot \sqrt{-4}$ because $\sqrt{-25}$ and $\sqrt{-4}$ do not define real numbers. $\sqrt{-25} \cdot \sqrt{-4} = i\sqrt{25} \cdot i\sqrt{4} = 5i \cdot 2i = 10i^2 = -10$

Note: *In this case, if you were to apply the Product Rule of Radicals you would get 10 instead of -10.*

Powers of *i*

$i = \sqrt{-1} = i$

$i^2 = -1$

$i^3 = i^2 i = -1i = -i$

$i^4 = i^2\, i^2 = (-1)(-1) = 1$

$i^5 = (i^2)^2(i) = (-1)^2 i = i$

$i^6 = (i^2)^3 = (-1)^3 = -1$

$i^7 = (i^2)^3(i) = (-1)^3 i = -i$

$i^8 = (i^2)^4 = (-1)^4 = 1$