

Solving a System of Three Equations



with THREE VARIABLES each

Example: $x + y + z = 2$
 $x - y + 2z = 5$
 $3x + 2y - z = 3$

- 1) Choose two of the 3 equations and add them to eliminate one of the variables...

$$\begin{array}{r} x + y + z = 2 \\ x - y + 2z = 5 \\ \hline 2x + 3z = 7 \end{array}$$

Box the equation

$2x + 3z = 7$

- 2) Next, choose two other equations (a different combination), and eliminate the same variable as you did in Step 1.

$$\begin{array}{r} x - y + 2z = 5 \\ 3x + 2y - z = 3 \end{array} \xrightarrow{\text{Multiply this by 2}} \begin{array}{r} 2x - 2y + 4z = 10 \\ 3x + 2y - z = 3 \\ \hline 5x + 3z = 13 \end{array}$$

Box this equation also

- 3) Take the two boxed equations and use them to eliminate another variable.

$$\begin{array}{r} 2x + 3z = 7 \\ 5x + 3z = 13 \end{array} \xrightarrow{\text{Multiply this by -1}} \begin{array}{r} 2x + 3z = 7 \\ -5x - 3z = -13 \\ \hline -3x = -6 \\ x = 2 \end{array}$$

- 4) Now that you found x, plug it back into one of the boxed equations to find another variable...

$$\begin{array}{l} 5x + 3z = 13 \\ 5(2) + 3z = 13 \\ 3z = 3 \\ z = 1 \end{array}$$

- 5) Plug the x and z into one of the original equations to find the last variable.

$$\begin{array}{l} x + y + z = 2 \\ 2 + y + 1 = 2 \\ y + 3 = 2 \\ y = -1 \end{array}$$

We find that $x = 2$, $y = -1$, and $z = 1$.

Solution: (2, -1, 1)

Solving a System of Three Equations

with a MISSING VARIABLE

Example: $a + b + c = 6$
 $a - b + 2c = 5$
 $- a \quad - c = -4$

1. Choose the two equations that contain the variable that is missing from the third equation, and eliminate that variable from those two equations.

$$\begin{array}{r} a + b + c = 6 \\ \underline{a - b + 2c = 5} \\ 2a + 3c = 11 \end{array}$$

2. Take the result from step 1 and pair it with the original equation that was missing a variable. Eliminate another variable...

$$\begin{array}{r} 2a + 3c = 11 \\ \underline{-a - c = -4} \end{array}$$

$$\begin{array}{r} \text{Multiply by 3} \rightarrow \begin{array}{r} -3a - 3c = -12 \\ \underline{2a + 3c = 11} \\ -a = -1 \end{array} \\ \text{Multiply } -a \text{ by } -1 \\ \text{to get } a \rightarrow \begin{array}{r} -a = -1 \\ \underline{-1} \\ a = 1 \end{array} \end{array}$$

3. Now that you found the value of a, plug it into one of the two-variable equations and solve for another variable...

$$\begin{array}{r} -a - c = -4 \\ -1 - c = -4 \\ \text{Multiply } -c \text{ by } -1 \\ \text{to get } c \rightarrow \begin{array}{r} -c = -3 \\ \underline{-1} \\ c = 3 \end{array} \end{array}$$

4. Plug the a and c into one of the original equations to find the last variable.

$$\begin{array}{r} a + b + c = 6 \\ 1 + b + 3 = 6 \\ b + 4 = 6 \\ \underline{b = 2} \end{array}$$

We find that $a = 1$, $b = 2$, and $c = 3$.

Solution: (1, 2, 3)