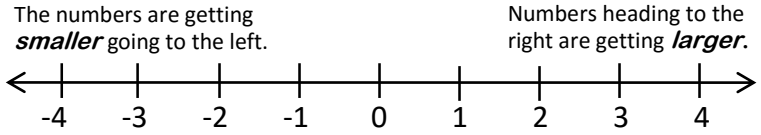
Graphing One Variable (& Compound) Inequalities

Consider the number line:



The arrows on the number line match the direction of the inequality symbols: Less than (<) faces the same way as the smaller numbers, and greater than (>) faces the same way as the larger ones.

Inequality Symbols

Greater than: >

For example, 13 > –4
Thirteen is greater than negative four.

Less than: <

For example, -3 < 9
Negative three is less than 9.

Greater than or equal to: ≥

For example, 6 ≥ 6
Six is greater than **or** equal to six.
6 is larger than 6 **or** 6 is equal to 6
(not true) (true)

Less than or equal to: ≤

For example, 9 ≤ 10
Nine is less than **or** equal to ten.
9 is less than 10 **or** 9 is equal to 10
(true) (not true)

Solving an Inequality

Solve as an equation: $-2x < -10$

$$\frac{-2x}{-2} > \frac{-10}{-2}$$

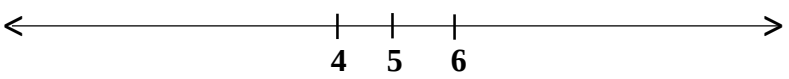
The solution: $x > 5$
x is greater than 5

Solve as you would any equation, but keep the inequality sign. Use normal rules of algebra, such as dividing both sides of the inequality by -2.

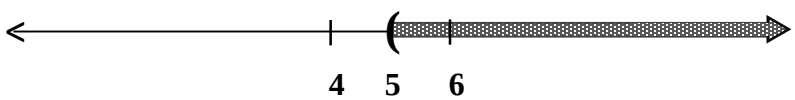
Note: When dividing or multiplying by a *negative* number only, the inequality symbol changes to the *opposite* symbol.

Graphing an Inequality

Graph the solution: **x > 5**



1. Place the number 5 on the number line.



2. Shade all values greater than 5.

We place a left parenthesis at 5, indicating that the value 5 makes the statement "x > 5" false. This and other graphing symbols are explained in the next portion of this handout.

Graphing One Variable (& Compound) Inequalities

Other Important Symbols

In some graphs, **>** is represented by

(or **○**

a left parenthesis or an open circle

In some graphs, **<** is represented by

) or **○**

a right parenthesis or an open circle

Left or right parentheses, or an open circle, tell us the value is not part of the solution set.

In some graphs, **≥** is represented by

[or **●**

a left bracket or a solid circle

In some graphs, **≤** is represented by

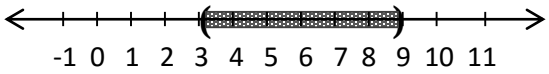
] or **●**

a right bracket or a solid circle

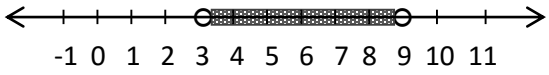
Left or right brackets, or a solid circle, tell us the value is included in the solution set.

Compound Inequalities

Sometimes, more than one inequality is described in the same expression. For example, when we want to say the solutions include all values between, but not including, 3 and 9, we could say that $x > 3$ and $x < 9$. A more compact expression for this is $3 < x < 9$. This *compound inequality* can be graphed in the following ways.

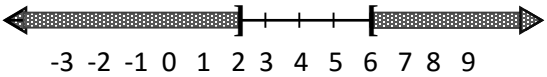


3 < x < 9 using parentheses

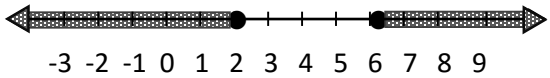


3 < x < 9 using open circles

If a compound inequality uses the word "or" instead of "and", the graph may have two shaded areas, representing the two parts of the expression. Here we graph $x \leq 2$ or $x \geq 6$.



x ≤ 2 or x ≥ 6 using brackets



x ≤ 2 or x ≥ 6 using solid circles

A Few Words About Interval Notation

Brackets and parentheses are also used in *interval notation*, which identifies the values in the solution set. For example, when $x > 3$ and $x < 9$, we would write that the solution set consists of the interval(3,9). As before, brackets or parentheses tell us whether the value is included in or excluded from the solution set. So if $x > -2$ and $x \leq 8$, the solution set would be written as (–2,8] in interval notation. This tells us that –2 is excluded and 8 is included.