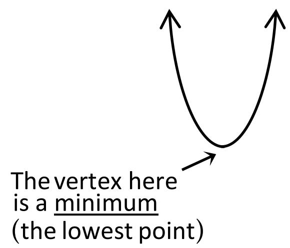
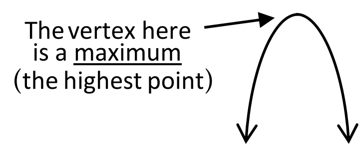
Finding the Vertex of a Parabola

Using the Vertex Formula: $x = -b/2a$

The vertex formula is one method for determining the vertex of a parabola. Recall that a parabola is formed when graphing a *quadratic equation*. The parabola will normally present with both ends heading up, or with both ends heading down, as seen below. To use the vertex formula, a *quadratic equation* must be put in the form $f(x) = ax^2 + bx + c$, where $a \neq 0$. We will use $f(x) = x^2 - 4x - 3$ as an example.





To find the coordinates of the vertex using $x = -\frac{b}{2a}$

1) Put the equation in proper form.

$$f(x) = x^2 - 4x - 3$$

2) Label the coefficients and the constant:

$$f(x) = \overset{a}{1x^2} - \overset{b}{4x} - \overset{c}{3}$$

We see that $a = 1$, and $b = -4$

3) Place the coefficients into the vertex formula: $x = \frac{-b}{2a}$

So, $x = \frac{-(-4)}{2\,(1)}$

$$x = \frac{4}{2}$$

$$x = 2$$

The x-coordinate of the vertex is 2.

4) To find the y-coordinate, first, change $f(x)$ to y.

$$f(x) = x^2 - 4x - 3 \rightarrow y = x^2 - 4x - 3$$

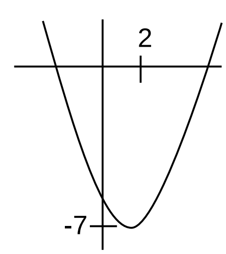
5) Then, plug the x value into the original function and solve for y.

$$y = (2)^2 - 4(2) - 3$$

$$y = 4 - 8 - 3$$

$$y = -7$$

The y-coordinate is -7, putting the vertex at (2, -7).



DAYTONA STATE COLLEGE
DaytonaState.edu/ASC
E-mail: ASC@DaytonaState.edu • Phone: (386) 506-3673

Finding the vertex of a parabola

Complete the Square Method

You may have used the Complete the Square method to *solve for x*. Sometimes we use complete the square to *find the vertex* of a parabola. We want to put the function into the *vertex form* of a quadratic function: $y = a(x - h)^2 + k$. In this example, the leading coefficient is a 1, as in $1x^2 - 10x + 21$:

$$f(x) = x^2 - 10x + 21$$

We have a trinomial, a polynomial with 3 terms. We can change the f(x) to y.

$$y = x^2 - 10x + 21$$

Divide the coefficient of the middle term by 2, and square it:

$$-10/2 = -5$$

$$(-5)^2 = 25$$

Add this quantity to each side of the equation.

$$y + 25 = x^2 - 10x + 21 + 25$$

Group the first two terms with the newly added quantity in a set of parentheses.

$$y + 25 = (x^2 - 10x + 25) + 21$$

Factor this trinomial grouping into its perfect square factors.

$$y + 25 = (x - 5)(x - 5) + 21$$

Simplify the expression, writing as a squared term with an exponent.

$$y + 25 = (x - 5)^2 + 21$$

Subtract (or add) to get y by itself.

$$y = (x - 5)^2 + 21 - 25$$

Simplify: $y = (x - 5)^2 - 4$

The function is now in the *vertex form* of a quadratic function: $y = a(x - h)^2 + k$

In this form you can determine the vertex (h, k), where h is the x-value of the vertex, and k is the y-value of the vertex. (The value a is the coefficient of the first term. In our example, a = 1.)

Our vertex is (5, –4). Watch the signs of the x- and y-values, so that you do not change the signs in the vertex formula.

When $a \neq 1$, the trick is to "factor out" the a, and be careful to add the correct quantity to each side of the equation. Try this example:

$$y = 3x^2 + 24x - 17$$

$$y = 3(x^2 + 8x) - 17$$

$$y + 3(16) = 3(x^2 + 8x + 16) - 17$$

$$y = 3(x + 4)(x + 4) - 17 - 3(16)$$

$$y = 3(x + 4)^2 - 65$$

$$h = -4,\ k = -65$$

The vertex is (–4, –65).

DAYTONA STATE COLLEGE
DaytonaState.edu/ASC
E-mail: ASC@DaytonaState.edu • Phone: (386) 506-3673